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Modelling and analysis of a symbiotic waste management system

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A municipal solid waste management system is symbiotic when there exists physical exchange of material or by-products between different treatment units. We propose a mathematical model for studying the interactive behaviour of different waste treatment operators in a symbiotic environment. Each operator is a self-interested entity, who sets his gate fee charge to maximise his own payoff. We study the properties and gate fee strategies of the operators, and also perform sensitivity analysis on various model parameters to discuss the local operator behaviour and the effects of various intervention strategies. We also propose a numerical algorithm to solve the model, yielding the optimal equilibrium gate fee charges, payoff and market share levels of different operators. Finally, computational studies based on a two-unit scenario in a case study of organic waste recycling is performed to demonstrate the interactive and dynamic behaviours of different operators. Our results strongly suggest that, to improve new treatment technology utilisation, subsidising the operating cost of the new treatment unit is more effective in the long-run than exerting control on the gate fee upper bounds of the operators by the system regulator. Furthermore, providing residual post-treatment discounts for treatment units can benefit the service users, rather than the waste treatment operators.

Keywords: sustainability; environmental management; modelling; symbiotic system; market competition

1. Introduction

Effective municipal solid waste (MSW) management continues to be an important priority in modern societies, the challenges of which include diminishing disposal space availability, harmful emissions, water and soil contaminations, and energy efficiency. In recent decades, advances in waste treatment and disposal technologies have significantly improved environmental and sustainability performance by enhancing the energy recovery efficiency and reducing hazardous emissions. These innovative MSW treatment technologies are now commercially available in various scales, offering promising alternatives to alleviate waste management problems, and also a source of renewable energy. For example in Singapore, the anaerobic digestion is used as an alternative of traditional incineration to treat organic waste more effectively, and recover biogas from the treatment process efficiently, which can then be used in power generation or as trucking fuel with less hazardous emission.

A common issue related to MSW treatment units is their secondary solid residues generated. Industrial symbiosis, as one of the key concepts of industrial ecology, offers a promising solution for alleviating these secondary pollution emissions by engaging traditionally separate units in a collective approach through physical exchange of materials, energy, water and by-products (Chertow 2007; Felicio et al. 2016). By operating in synergy, these units usually can achieve a collective environmental and economic benefit that is greater than the sum of their individual benefits achieved by working alone. Therefore, there is an increasing interest in integrating and supplementing the incumbent MSW treatment systems with different treatment units based on the principle of industrial symbiosis to collectively improve performance more effectively. In the literature, several works have proposed operations management methods to support the analysis and design of such systems. For example, Chen et al. (2014) proposed an inexact chance-constrained programming model by incorporating inventory theory for planning the waste allocation batch and waste transferring period with different constraint violation risks in a symbiotic waste management system. Zhang, Huang, and He (2014) built a linear chance-constrained programming model to design the transportation and inventory scheme for each collection, distribution and disposal facility in a waste management system with multi-echelon supply chains. Inghels, Dullaert, and Vigo (2016) presented a dynamic tactical planning model for minimising the transportation costs of a given symbiotic waste handling and processing system composed of multiple collection and treatment facilities. Xiong, Ng, and Wang (2016) developed a two-stage mixed-integer stochastic programming model with incorporating the life-cycle assessment approach into the symbiotic waste management system design process

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for comprehensively optimising the system's economic, energy and environmental performance. More applications can be seen in Zhang, Huang, and He (2011) and Chen et al. (2016).

MSW management is conventionally regarded as a public service, with the local government or authority having sole proprietorship of all entities and resources for the end-to-end activities from waste collection to final disposal. More recently, however, private sector participation in MSW management is increasingly being studied and applied to help reduce public overheads, improve service quality, motivate private investment and introduce innovative technologies in many countries (Suocheng, Tong, and Wu 2001). For example, the liberalisation of waste collection service has been used to promote the competition for improving the service quality in Singapore (Bai and Sutanto 2002). Besides, the environmental policy-makers of Singapore are proposing a feasibility study of improving food waste recycling service by introducing anaerobic digestion units in the townships (En 2015). Independent and qualified private operators can enter the market as digestion unit operators and offer their treatment services to the waste collectors.

In MSW treatment systems with private sector participation, a question of policy interest is whether the different, self-interested treatment operators (comprising of possibly different treatment technologies) can co-exist in an economically feasible and sustainable manner during their competition. This is of importance for stakeholders, as past incidents have revealed that new waste treatment initiatives can fail due to under-utilisation issues during implementation, e.g. the case of food waste recycler IUT Global in Singapore (The Straits Times 2011). In particular, when the new treatment service provider is unable to divert and sustain a reasonable level of input feedsock from the incumbent operators to maintain a healthy cashflow position, they can face severe financial deficits, leading to shutdowns, since many such new treatment units constitute extremely expensive capital investments. The system regulator would also like to discover the effects of implementing various policy interventions to improve the system performance. For instance, the regulator can sometimes impose restrictions on gate fee charges to alleviate service users' economic burden, or introduce some form of financial incentives or subsidies to the new treatment operators, in order to promote their implementation (World Bank 2000). In such cases, the issues of interest would be the reacting strategies of the other treatment operators, and also the resulting behaviour of the system as a whole.

Relatively few works have studied market competition in deregulated waste management systems. Davila, Chang, and Diwakaruni (2005) proposed a two-tiered gray integer programming-based game theory approach to help scrutinise scenarios wherever landfills display competitive behaviour under an increasing need for their services. The first tier is a gray integer programming model for minimising the net system cost over one year planning period by routing waste streams to two local landfill plants. The second tier is a series of gray integer submodels with different landfill tipping fee strategies among competition for understanding the minute effect of landfill tipping fee changes on a city's optimal routing decision. Jørgensen (2010) studied a differential game among three neighbouring regions in a finite time horizon, in which each region decides to dump a fraction of its waste stock to other regions for reducing the cost incurred by holding waste stock. Bárcena-Ruiz and Casado-Izaga (2015) studied a two-stage game between two private collection firms, who pursue maximal payoffs by deciding their locations (first-stage decision) and prices (second-stage decision) when the government requires private collection firms or consumers to bear the waste transportation fees. It showed that regardless of whether two private collection firms' locations are chosen simultaneously or sequentially, the optimal location of waste collection point for maximising weighted welfare of firms and consumers is in the middle of city. Besides, the optimal location of collection point is outside the middle of city only when the transportation fees are paid by consumers and the firms choose their locations sequentially.

Although a number of operations management models have been proposed for studying and optimising waste management activities (see Darlington and Rahimifard 2007; Staikos and Rahimifard 2007), to the best of the authors' knowledge, no prior work has proposed a model to study the behaviour of private MSW treatment unit operators in a symbiotic system under a market competitive gate fee charge environment. This research is of significant importance in the context of implementing market competition in the modern MSW management industry effectively. Besides, as remarked in Karmperis et al. (2013), a decision support model identifying the strategy that forms an equilibrium state could help enhance the sustainability of the entire MSW management system. However, no published work has applied the market mechanism and equilibrium to form a sustainable symbiotic waste management system with private sector competition.

We consider in this work a symbiotic waste management system, where self-interested private treatment service providers (also termed *operators*) can compete in the MSW treatment market by setting their preferred gate fee charge levels to maximise their individual operational payoffs. On the other hand, the treatment service *users* (e.g. kerbside waste collectors) can choose their preferred service providers based on the price attractiveness. The operators interact with each other through gate fee competition for the market share of available input feedstock, and possibly also through co-treatment. In particular, we assume that some operators may require the service of other units to process their post-treatment residuals. Therefore, the relationships among the operators are not only competitive, but also *symbiotic* in nature.

In this paper, we model the symbiotic waste management system to study the qualitative behaviour of the treatment unit operators. The preference of treatment service users depends on the on gate fee charges and is modelled using a utility-based market share model, which is widely applied in the literature (see Adler, Pels, and Nash 2010; Bernstein and Federgruen 2004; Goodale, Verma, and Pullman 2003). The model is solved by a proposed iterative algorithm to yield the optimal equilibrium conditions, such as the respective gate fee charges, payoff levels and market share levels. The term *equilibrium* in this study refers to the state in which no operator has positive marginal payoffs by changing its gate fee charge level. We also perform sensitivity analysis on various model parameters to obtain descriptions of the interactive behaviour between the operators. Such analysis, although mostly qualitative in value, can help elucidate a clearer depiction of the system interactions, and serve to sharpen the intuition of policy-makers in developing effective waste management policies for achieving environmental sustainability aspirations.

The rest of the paper is organised as follows. Section 2 presents the problem description and modelling framework. Section 3 conducts a comprehensive analysis on the interactive behaviour of waste management unit operators and various system parameters. Section 4 proposes a central-control model for the market competition and central-control method comparison. Section 5 demonstrates the computational studies to support decision-makings of establishing a sustainable waste management system with market competition in Singapore. Section 6 concludes this paper and recommends future research directions.

2. Problem and model description

In this section, we describe the details and model formulations of the problem of study. Figure 1 shows the overview of the modelling framework in this study, the four steps of which are summarised as follows:

(1) At the Model Preparation step, the waste management process is first described to provide background information of the studied problem. The system boundary and assumptions are then determined for model formulation.
(2) At the Model Formulation step, the utility-based market share model is adopted to form the payoff function and payoff maximisation model for each waste treatment unit operator in the system.
(3) At the Model Analysis step, the optimal gate fee strategy for each treatment unit operator is analysed first based on his payoff function. A comprehensive sensitivity analysis on various model parameters is then conducted to depict the interactive behaviour of different treatment unit operators and develop policies for improving system's environmental sustainability performance at some initial system state.
(4) At the Model Application step, the proposed model is applied to simulate the market competition process, validate the qualitative results from the Model Analysis step, and discuss the impact of different intervention policies on the system performance at the optimal equilibrium state.

2.1 *Problem description and background*

The general process of MSW management is as follows. Mixed MSWs are first collected and presorted by refuse collectors. Next, material recovery takes place for recyclable and re-usable items. The remaining wastes are then sent to, and treated via various approaches (such as waste-to-energy) for volume compression and useful energy recovery. Residuals generated may then require further post-treatment. Finally, all post-treatment residuals and other un-treatable components are disposed in landfill sites.

In this work, we focus on the MSW treatment stage of the process as illustrated in Figure 2, which comprises different technologies and units that can treat either specific streams of MSW (e.g. anaerobic digestion for the treatment of organic waste), or any MSW stream (e.g. incineration). The collected and presorted waste streams are collectively referred to as the *input feedstock*. The treatment service users (e.g. refuse collectors) forward the input feedstock to one or several of the treatment units, and pay a service tipping fee, termed as the *gate fee*, to the treatment unit operator. We also refer to treatment units that treat only specific MSW stream components as specialised treatment units (STUs), and those that can treat any MSW stream component as general treatment unit (GTU). The commonly implemented STU includes aerobic composting, anaerobic digestion and gasification, which convert organic and biomass wastes into fuel gases and composts by different biodegradable and thermal treatment technologies. The GTU in our context refers to incineration, which burns wastes as fuel with high volume of air to generate electricity. More detailed information about the unit selection criteria for different waste streams can be found in Xiong, Ng, and Wang (2016). In particular, the market competition model is built on the following assumptions:

(1) The MSW streams considered in the input feedstock can be treated by all units in the system, so that the boundary of analysis focuses on MSW stream components that are of interest to governing agencies, e.g. organic waste.

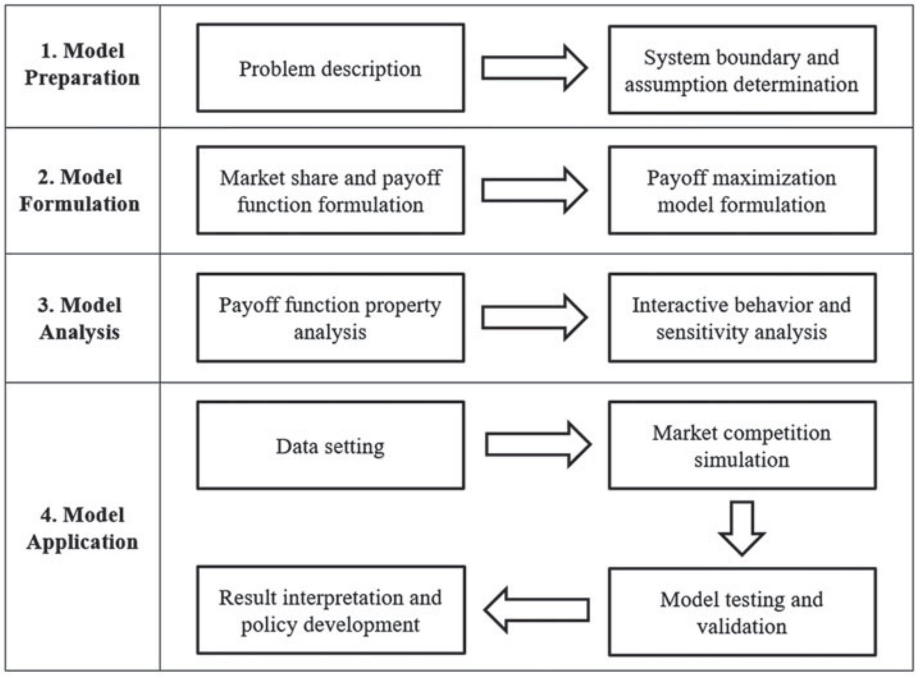


Figure 1. Overview of the modelling framework applied in this study.

(2) Only one GTU is considered in the studied system. This is because the GTU like incineration generally exhibits significant economies of scale, typically favouring large scale, and requires a stable waste feedstock supply (Miranda and Hale 1997). This fact in turn let a GTU require a large catchment area, in which several STUs could serve.
(3) The STUs can produce post-treatment residual that requires further processing by the GTU before its final disposal. For instance, solid residual from the anaerobic digestion process must be further post-treated (e.g. via combustion) before it is permitted for landfilling disposal.
(4) Each treatment unit operator can freely determine his gate fee charge level, subjected to an upper bound imposed by state regulation, to maximise his own payoff. The gate fee is an important component of the revenue stream of operator, and also a key instrument for the operator to make his treatment service economically attractive to users.
(5) The landfilling process and the post-treatment process for airborne and waterborne emissions from all treatment units are excluded in the model. The system's environmental performance will be considered in an indirect way by discussing the policy interventions for adjusting the environmental-friendly STU's market share.

2.2 *Model description*

The notation used in the model formulations are listed in the Table 1. The index value of '0' denotes the GTU. Other notation are introduced as required later in paper.

We first introduce the market share function $S_f(\boldsymbol{\eta})$ to model the proportion of input feedstock received by the various treatment units, where $f \in \mathcal{F}$. The market share function is based on the *multinomial logit demand* model commonly used in revenue management and marketing literature (Besanko, Gupta, and Jain 1998; Guadagni and Little 1983). The attractive feature of such model is that it can model customer preferences with different characteristics (e.g. gate fee) of a treatment unit operator, so that his operational payoff function can be formulated. In our model, the market share of input feedstock received by treatment unit f is given by

$$S_f(\boldsymbol{\eta}) = \frac{e^{\beta_f - \alpha_f \eta_f}}{\sum_{f \in \mathcal{F}} e^{\beta_f - \alpha_f \eta_f}}, \ \forall f \in \mathcal{F}, \tag{1}$$

where $\alpha_f > 0$ and β_f are parameters defining the function $\beta_f - \alpha_f \eta_f$ associated with treatment unit f, which can be regarded as a first-order approximation or surrogate for the 'attractiveness' of using treatment unit f. Clearly, this is non-increasing in

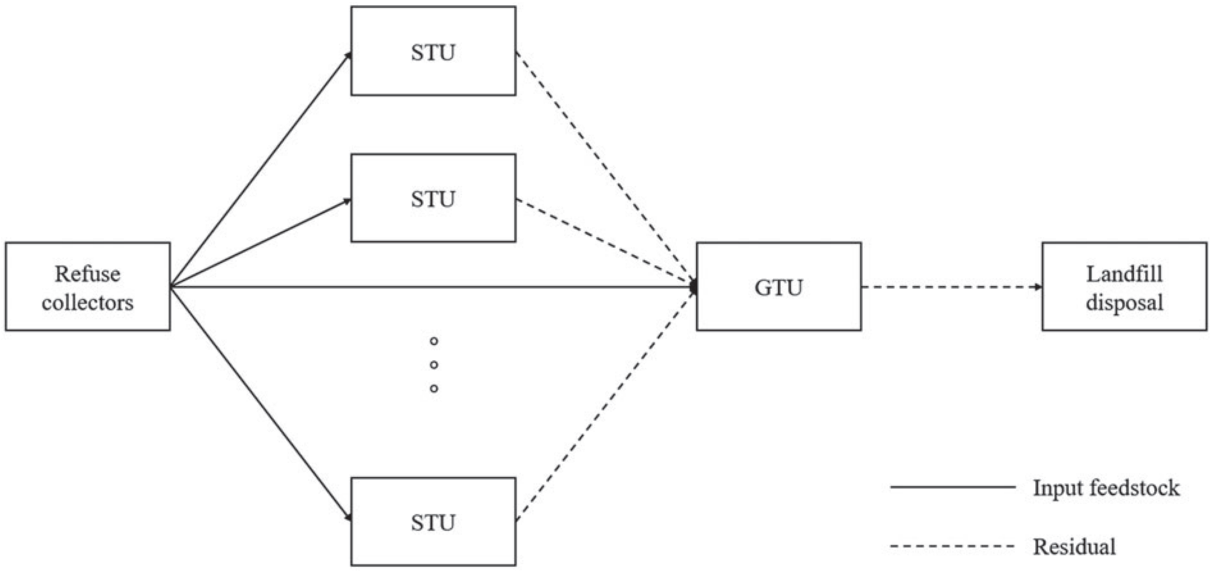


Figure 2. Topology diagram of the considered MSW treatment process.

Table 1. Notations used in the model formulation.

Notation	Definition
$\mathcal{I}$	The set of STU, where $\mathcal{I} = \{1, 2, \ldots, m\}$
$\mathcal{F}$	The set of all treatment units, where $\mathcal{F} = \{0\} \cup \mathcal{I}$
η_f	Gate fee of the treatment unit f, where $f \in \mathcal{F}$
$\boldsymbol{\eta}$	Gate fee vector, i.e. $\boldsymbol{\eta} = (\eta_0, \eta_1, \ldots, \eta_m)$
$\eta_f^{\dagger}$	Optimal gate fee of the treatment unit f, where $f \in \mathcal{F}$
η_f^{∞}	Optimal equilibrium gate fee of the treatment unit f, where $f \in \mathcal{F}$
$\overline{\eta}_f$	Gate fee upper bound of the treatment unit f, where $f \in \mathcal{F}$
μ_f	Operation profit rate of the treatment unit, where $f \in \mathcal{F}$
δ_f	Residual coefficient of the treatment unit, where $f \in \mathcal{F}$
α_f	Customer price sensitivity to the treatment unit f, where $f \in \mathcal{F}$
β_f	Other comprehensive customer utility index of the treatment unit f, where $f \in \mathcal{F}$
ω	Total volume of collected input feedstock
$\underline{S_f}$	Desired market share lower bound of the treatment unit f, where $f \in \mathcal{F}$
$\underline{P_f}$	Desired payoff lower bound of the treatment unit f, where $f \in \mathcal{F}$
$S_f(\boldsymbol{\eta})$	Market share function of the treatment unit f, where $f \in \mathcal{F}$
$P_f(\boldsymbol{\eta})$	Payoff function of the treatment unit f, where $f \in \mathcal{F}$

the gate fee level η_f. We denote $\boldsymbol{\eta}_{f-}$ as the gate fee vector $\boldsymbol{\eta}$ without the fth component η_f for all $f \in \mathcal{F}$. For each treatment operator f, it is also easy to see that his market share $S_f(\eta_f|\boldsymbol{\eta}_{f-})$ is non-increasing in his gate fee η_f for all $f \in \mathcal{F}$.

In the system considered, the STUs accrue revenues through the collection of gate fees for receiving input feedstock, as well as by selling recovered products such as power or energy (e.g. electricity or trucking fuel) from the treatment process. Its expenditures include variable treatment operation costs (i.e. transportation, process and post-treatment cost) and also the cost for using the service of GTU for disposing its post-treatment residuals. This last component is essentially the gate fee charged by the GTU. For convenience, we denote μ_f as the operation profit rate from recovered product sales per unit less all cost components except for the residual post-treatment cost incurred as the form of gate fee payment to the GTU. Therefore, the variation of any operation revenue and cost component during the market competition in each unit can be captured by the variation of its parameter μ_f. Also, any factor (e.g. economies of scale) affecting the operation revenue and cost component can be indirectly considered by varying the parameter μ_f during the competition process. The payoff maximisation model for an STU (namely $\forall f \in \mathcal{I}$) is then defined as follows.

$$\max_{\eta_f} \quad \omega S_f\left(\eta_f|\boldsymbol{\eta}_{f^-}\right)\left(\eta_f+\mu_f-\delta_f\eta_0\right) \tag{2}$$

$$\text{s.t.} \quad \eta_f+\mu_f-\delta_f\eta_0>0 \tag{3}$$

$$\eta_f \leqslant \overline{\eta}_f \tag{4}$$

Objective (2) is the payoff function of the STU f, computed using the product of the total input feedstock received, $\omega S_f\left(\eta_f|\boldsymbol{\eta}_{f^-}\right)$, where ω is the total volume of input feedstock collected, and profit rate $\eta_f+\mu_f-\delta_f\eta_0$. The parameter $\delta_f \in (0, 1)$ denotes the *residual coefficient*, which is the proportion per unit input feedstock received that requires post-treatment by the GTU. Constraint (3) ensures that the operator is only interested in positive marginal profits. Constraint (4) implies the gate fee of each STU should be less than a regulated upper bound. This gate fee upper bound is imposed to inhibit the socially unacceptable prices in MSW services.

For the GTU, the operator's revenue stream comes from gate fees collected from both classes of service users, i.e. the refuse collectors, and the STUs, as well as selling recovered energy products. The considered expenditures of GTU include variable treatment operation costs and also final disposal service costs (i.e. the landfilling disposal cost). Similarly, we define its operation profit rate μ_0 as the recovered energy product sales per unit less than all cost components. And the variation of any operation revenue and cost component caused by any factor in the GTU can be reflected by the variation of μ_0. The payoff maximisation model for the GTU is stated as follows.

$$\max_{\eta_0} \quad \left[\omega S_0\left(\eta_0|\boldsymbol{\eta}_{0^-}\right)+\sum_{f\in\mathcal{I}}\omega S_f\left(\eta_0|\boldsymbol{\eta}_{0^-}\right)\delta_f\right](\eta_0+\mu_0) \tag{5}$$

$$\text{s.t.} \quad \eta_0+\mu_0>0 \tag{6}$$

$$\eta_0 \leqslant \overline{\eta}_0 \tag{7}$$

Objective (5) is the payoff function of the GTU, which is computed by the product of total input feedstock from both refuse collectors and STUs, i.e. $\omega S_0\left(\eta_0|\boldsymbol{\eta}_{0^-}\right)+\sum_{f\in\mathcal{I}}\omega S_f\left(\eta_0|\boldsymbol{\eta}_{0^-}\right)\delta_f$, and the profit rate $\eta_0+\mu_0$. Constraint (6) ensures the GTU has a positive marginal profit. Constraint (7) requires the gate fee of GTU to satisfy the corresponding government regulations as Constraint (4).

By simulating the market competition process (using approaches for example in Hsu, Lee, and Liao 2010; Wang, Meng, and Zhang 2014; Xing and Wu 2001), a straightforward way to apply the models (2)–(4) and (5)–(7) is as follows. At the beginning of each stage, each treatment unit operator observes his competitors' gate fees from the market. Each operator then applies and solves the proposed payoff maximisation model to evaluate his current payoff and yield a new optimal gate fee. At the end of each stage, all operators adjust and updated their gate fees. This process iterates until the system approaches equilibrium, i.e. no operator has incentive to adjust his gate fee as no marginal payoff gains can be gained. Economic feasibility of the system is then evaluated by the decision-makers, by checking if the payoffs achieved by individual operators are acceptable. For instance, if the optimal payoff of a unit operator does not even cover his fixed cost repayments, then the treatment unit cannot sustain economically. This also implies that the overall system, as it is, cannot be economically feasible. In this paper, however, the major focus is to analyse and to understand the properties and behaviour of MSW treatment operators and system. Such analysis, although mainly qualitative in nature, can yield useful insights for decision-makers, regardless of the actual parameter values used.

3. Model properties and sensitivity analysis

In this section, we study in detail the optimal gate fee strategies and interactive behaviours of STU and GTU during competition by using analytical means. We first study the payoff functions' properties for the two types of treatment units. We show that in general, treatment units can exhibit different dominant operating regimes, depending on their gate fee levels. They may choose to operate in a market competition regime, by depressing gate fee charge to increase primary feedstock share. They can also exit the competition and focus on the residuals feedstock market. To demonstrate these properties more clearly, we consider a two-unit system (one STU and one GTU) and provide illustrations based on the real case of Singapore. Moreover, we perform a comprehensive sensitivity analysis on the interaction between the STU and GTU, and discuss its application in mitigating environmental sustainability issues for both the private operators and the system regulator.

To simplify the notation, we denote the payoff functions as follows:

$$P_f(\boldsymbol{\eta})=\omega S_f(\boldsymbol{\eta})(\eta_f+\mu_f-\delta_f\eta_0), \quad \forall f\in\mathcal{I}, \tag{8}$$

$$P_0(\boldsymbol{\eta}) = \left[\omega S_0(\boldsymbol{\eta}) + \sum_{f\in\mathcal{I}} \omega S_f(\boldsymbol{\eta})\delta_f\right](\eta_0 + \mu_0). \tag{9}$$

In addition, we denote $\underline{\eta_f} = -\mu_f + \delta_f \eta_0$ for all $f \in \mathcal{I}$, and $\underline{\eta_0} = -\mu_0$. That is, $\underline{\eta_f}$ is the lower bound of the gate fee for treatment unit f to secure a positive marginal payoff, where $f \in \mathcal{F}$. We also denote $\eta_f^\dagger$ as the optimal gate fee and denote η_f^∞ as the optimal equilibrium gate fee, where $f \in \mathcal{F}$. Specifically, $\eta_f^\dagger$ is the gate fee that maximises the payoff function $P_f(\boldsymbol{\eta})$ over the domain $(\underline{\eta_f}, \overline{\eta}_f]$, and η_f^∞ is the $\eta_f^\dagger$ at the equilibrium condition. Finally, we denote $z_f = e^{\beta_f - \alpha_f \eta_f}, \forall f \in \mathcal{F}$.

3.1 *Properties of the STU's payoff function*

We first consider the payoff function of the STU, and its optimal gate fee strategy.

Proposition 3.1 *The payoff function $P_f(\eta_f|\boldsymbol{\eta}_{f^-})$ is log-concave with respect to η_f when $\eta_f > \underline{\eta_f}$ for all $f \in \mathcal{I}$. Furthermore, the payoff maximising gate fee η_f^* over the domain $(\underline{\eta_f}, \infty)$ can be obtained by solving the following:*

$$\eta_f^* - \underline{\eta_f} - \frac{z_f^* + \sum_{f'\in\mathcal{F}/\{f\}} z_{f'}}{\alpha_f \sum_{f'\in\mathcal{F}/\{f\}} z_{f'}} = 0, \quad \textit{where } z_f^* = e^{\beta_f - \alpha_f \eta_f^*}. \tag{10}$$

All technical proofs in this paper are provided in the Appendix 1. Proposition 3.1 shows that there must exist an unique gate fee η_f^* for each STU to achieve its maximum payoff over the domain $(\underline{\eta_f}, \infty)$. The value of η_f^* can be obtained by numerical solvers. Regarding the impact of the gate fee upper bound $\overline{\eta}_f$ on the optimal gate fee $\eta_f^\dagger$ in a STU, Proposition 3.1 implies that, if $\eta_f^* \leqslant \overline{\eta}_f$, then $\eta_f^\dagger = \eta_f^*$. Otherwise $\eta_f^\dagger = \overline{\eta}_f$.

Corollary 3.2 *For every $f \in \mathcal{I}$, the payoff maximising gate fee η_f^* over the domain $(\underline{\eta_f}, \infty)$ is positively correlated with the $\eta_{f'}$, where $f' \in \mathcal{F}/\{f\}$, and is also positively correlated with the residue coefficient δ_f. It is negatively correlated with the operation profit rate μ_f.*

The above describes the action of a STU f in adjusting its gate fee setting for maximising payoff over the domain $(\underline{\eta_f}, \infty)$ in response to gate fee changes of other units. Furthermore, since the gate fee lower bound $\underline{\eta_f}$ is defined as $\underline{\eta_f} = -\mu_f + \delta_f \eta_0$, Corollary 3.2 implies that η_f^* is positively correlated with the $\underline{\eta_f}$, which can be regarded as the negated profit rate of the unit without collection of gate fees.

Next, we consider a two-unit system, in which there is one STU (denoted as Unit 1) and one GTU (denoted as Unit 0). The following corollary is obtained in such scenario.

Corollary 3.3 *The log-payoff function* $\ln P_1(\eta_0, \eta_1^*)$ *is positively correlated with the GTU's gate fee η_0 when $\delta_1 < \alpha_0/\alpha_1$, and is negatively correlated with η_0 when $\delta_1 > \alpha_0/\alpha_1$.*

The Corollary 3.3 implies that in the two-unit scenario, if the residual coefficient of STU δ_1 is less than the threshold value α_0/α_1, then both its payoff maximising gate fee η_1^* over the domain $(\underline{\eta_1}, \infty)$ and corresponding log-payoff $\ln P_1(\eta_0, \eta_1^*)$ are increasing in the GTU's gate fee η_0. This is intuitive, since if the unit can improve net profits through reducing residuals generated, it is then able to sustain some market share losses while commanding a higher gate fee for its service. Besides, if δ_1 is larger than the threshold α_0/α_1, then the decrease of η_0 will increase the log-payoff $\ln P_1(\eta_0, \eta_1^*)$. It implies that when the residual coefficient is relatively high in the STU, then the impact of residual post-treatment cost can be significant for its profitability.

Example 1 We illustrate an example of a two-unit scenario, in which the GTU is assumed to be an incineration unit, and the STU is assumed to be an anaerobic digestion unit. The operation profit and cost data of such units in Singapore are applied. Specifically, we set $\mu_0 = -6.67$ SGD/ton and $\mu_1 = -53.63$ SGD/ton (McCrea et al. 2009). We set the residual coefficient $\delta_1 = 0.31$ according to De Bere (2000), and $\delta_0 = 0$ as the incineration does not generate residues requiring post-treatment, except for the final disposal (i.e. ashes landfilling). Moreover, we assume that the multinomial logit model parameters are $\alpha_0 = \alpha_1 = 0.1$, and $\beta_0 = \beta_1 = 15$ for simplicity. Because the derived insights from the analysis on treatment unit operators' payoff functions are based on mathematical properties, they hold even if these data assumptions are changed. Finally, we consider the organic MSWs as the input feedstock, and set $\omega = 1,409,400$ tons based on Singapore's data in 2014 (NEA 2013).

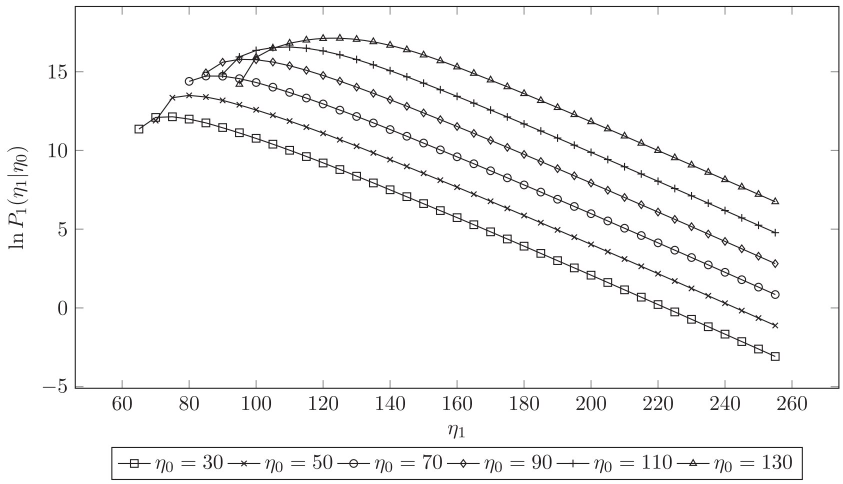


Figure 3. The curves of $\ln P_1(\eta_1|\eta_0)$ with different η_0 in the two-unit scenario.

Figure 3 shows the STU's log-payoff function $\ln P_1(\eta_1|\eta_0)$ for the two-unit example, for different GTU gate fees η_0, based on the data in the Example 1. It can be noted that when η_0 increases, both the η_1^* and $\ln P_1(\eta_0, \eta_1^*)$ will increase, which is consistent with the results in Corollaries 3.2 and 3.3.

3.2 *Properties of the GTU's payoff function*

We next consider the payoff function of the GTU and its optimal gate fee strategy. Unfortunately, unlike the case of STU, the payoff function of GTU is not log-concave.

PROPOSITION 3.4 *Define the function $U(\boldsymbol{\eta})$ as*

$$U(\boldsymbol{\eta}) = \ln \sum_{f \in \mathcal{F}} z_f + \ln \sum_{f \in \mathcal{F}} z_f \delta_f - \ln z_0 - \ln(\eta_0 - \underline{\eta_0}) - \ln \alpha_0 \sum_{f \in \mathcal{I}} z_f (1 - \delta_f), \tag{11}$$

and the minimal point $\tilde{\eta}_0$ with $\partial U(\tilde{\eta}_0|\boldsymbol{\eta}_{0-})/\partial \eta_0 = 0$. So if $U(\tilde{\eta}_0|\boldsymbol{\eta}_{0-}) \geqslant 0$, the GTU's optimal gate fee $\eta_0^\dagger$ should be set to its upper bound $\overline{\eta}_0$. Otherwise, the $\eta_0^\dagger$ should be set to: (a) $\overline{\eta}_0$, when $\overline{\eta}_0 \in (\underline{\eta_0}, \eta_0^-] \cup (\eta_0^\Gamma, \infty)$; (b) η_0^-, when $\overline{\eta}_0 \in (\eta_0^-, \eta_0^\Gamma)$; (c) η_0^- or $\overline{\eta}_0$, when $\overline{\eta}_0 = \eta_0^\Gamma$. The η_0^- is the solution of model (12):

$$\min_{\eta_0 \in (\underline{\eta_0}, \infty)} \eta_0, \quad \text{s.t. } U(\eta_0|\boldsymbol{\eta}_{0-}) \leqslant 0, \tag{12}$$

and η_0^Γ is the solution of model (13):

$$\min_{\eta_0 \in (\eta_0^-, \infty)} \eta_0, \quad \text{s.t. } \ln P_0(\eta_0|\boldsymbol{\eta}_{0-}) \geqslant \ln P_0(\eta_0^-|\boldsymbol{\eta}_{0-}). \tag{13}$$

Proposition 3.4 provides the optimal gate fee strategy for the GTU, given the STUs' gate fee settings. If $U(\tilde{\eta}_0|\boldsymbol{\eta}_{0-}) \geqslant 0$, the payoff function of GTU is always non-decreasing, and hence it is always optimal to set its gate fee as high as possible, i.e. set $\eta_0^\dagger = \overline{\eta}_0$. More interestingly, if $U(\tilde{\eta}_0|\boldsymbol{\eta}_{0-}) < 0$, we can obtain a log-payoff function similar to the one in the two-unit scenario illustrated in Figure 4. It is observed that there are three important points: local maximum η_0^-, local minimum η_0^+ and η_0^Γ with $\ln P_0(\eta_0^-|\boldsymbol{\eta}_{0-}) = \ln P_0(\eta_0^\Gamma|\boldsymbol{\eta}_{0-})$. Therefore, the optimal gate fee $\eta_0^\dagger$ depends on which interval the specific level of the upper bound $\overline{\eta}_0$ is in: $(\underline{\eta_0}, \eta_0^-]$, $(\eta_0^-, \eta_0^\Gamma)$, or $[\eta_0^\Gamma, \infty)$. In particular, when the η_0 is sufficiently low, we observe a dominant market competition regime, characterised by the η_0^+ and log-concavity of the payoff function, which is very similar to the case of STU. On the other hand, when the η_0 is high, the operator exits the market competition regime, and profit margins are now dominated by treatment of residues from STUs. Because the STU has no alternative but to use the GTU's service for post-treating residues, the GTU can always maximise his benefits by charging a high premium for the post-treatment service. Furthermore, recall from Corollary 3.2 that increasing η_0 leads the STUs to further increase their η_f^*. This property reveals that, it is necessary for the regulator to mandate a sensible upper bound on the gate fee charge to prevent escalating

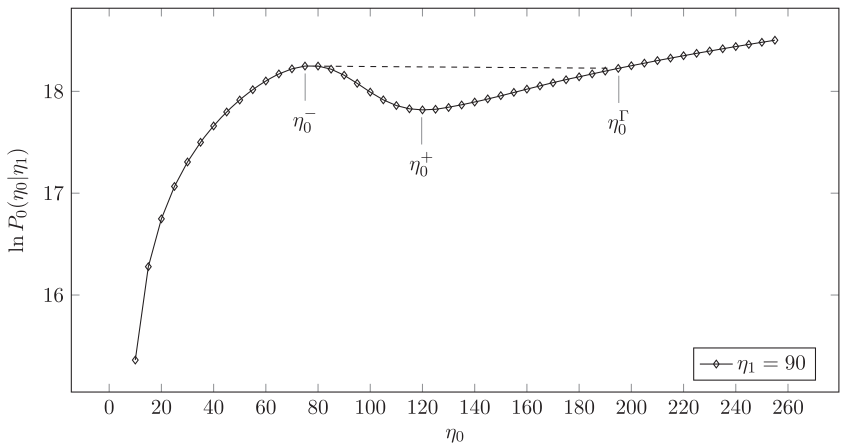


Figure 4. The curve of $\ln P_0(\eta_0|\eta_1)$ when $\eta_1 = 90$ in the two-unit scenario.

costs of waste treatment. Without such upper bound, the optimal gate fee could escalate to unacceptably high levels, i.e. $\eta_0^\dagger \in [\eta_0^\Gamma, \infty)$ and $\eta_0^\dagger \to \infty$. On the other hand, the GTU might not necessarily always choose the upper bound imposed as its optimal gate fee. In particular, whenever $\overline{\eta}_0 \in (\eta_0^-, \eta_0^\Gamma)$, the operator prefers a strictly lower gate fee η_0^-. Therefore, Proposition 3.4 provides useful information for the regulator to choose an appropriate upper bound.

We further discuss the influence of gate fee of STU on the payoff of GTU.

COROLLARY 3.5 *In the two-unit scenario, the log-payoff function* $\ln P_0(\eta_1|\eta_0)$ *is increasing in the STU's gate fee* η_1.

The Corollary 3.5 indicates that given the GTU's gate fee η_0, the gain in its payoff by obtaining more market share due to the increase of η_1 exceeds the loss in its payoff by treating less post-treatment residues from the STU. This is because the value of δ_1 is lower than 1. Therefore, the increase of η_1 leads the GTU to actually receive more total amount of incoming feedstock.

The following results give two special cases in which the GTU's optimal gate fee calculation process can be simplified.

COROLLARY 3.6 *The payoff function* $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ *is log-concave with respect to* η_0, *when* $\underline{\eta_0} < \eta_0 \leqslant \frac{1}{\alpha_0}[\beta_0 - \ln(\sum_{f\in\mathcal{I}} z_f - r\sum_{f\in\mathcal{I}} z_f\delta_f) + \ln(r-1)]$, *where* $r = \sqrt{\sum_{f\in\mathcal{I}} z_f / \sum_{f\in\mathcal{I}} z_f\delta_f}$.

Corollary 3.6 implies that, if $\underline{\eta_0} < \overline{\eta}_0 \leqslant \frac{1}{\alpha_0}[\beta_0 - \ln(\sum_{f\in\mathcal{I}} z_f - r\sum_{f\in\mathcal{I}} z_f\delta_f) + \ln(r-1)]$, then the $\eta_0^\dagger$ can be calculated by simply letting the first-order derivative of $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ be equal to zero. Actually, according to the proof of Proposition 3.4, the η_0^- must be the $\eta_0^\dagger$ under this condition. Hence, we can calculate the $\eta_0^\dagger$ by simply solving the model (12).

COROLLARY 3.7 *In the two-unit scenario, if* $U(\tilde{\eta}_0|\eta_1) < 0$, *then* $\eta_0^\Gamma < \eta_0^-/\delta_1$.

The Corollary 3.7 indicates that if the condition $\overline{\eta}_0 \geqslant \eta_0^-/\delta_1$ holds in the two-unit scenario, then $\overline{\eta}_0$ must be the $\eta_0^\dagger$ according to the proof of Proposition 3.4.

3.3 *Sensitivity analysis with application to environmental sustainability*

Innovative specialised treatment techniques such as anaerobic digestion for organic waste are known to outperform conventional treatment technique such as incineration, in terms of enhancing energy recovery and reducing hazardous emissions (see, e.g. Khoo, Lim, and Tan 2010). However, these new treatment units may not be able to achieve sufficient utilisation to sustain economically, or divert enough waste stream from conventional treatment units to improve environmental performance. Therefore, the main hurdle and challenge of achieving environmental sustainability target is usually in implementing these new treatment units in a practically feasible manner, i.e. financially sustainable manner. In the following we present a sensitivity analysis on the gate fee and market share behaviour of different treatment units. The objective of the study is to elucidate the interaction effect of the treatment units about some initial system state, so as to evaluate the effect of actions such as gate fee adjustments. Due to the problem complexity, this sensitivity is only conducted in a two-unit scenario (i.e. one STU and one GTU) as limited analytical results and intuition can be obtained beyond this basic case. However, since

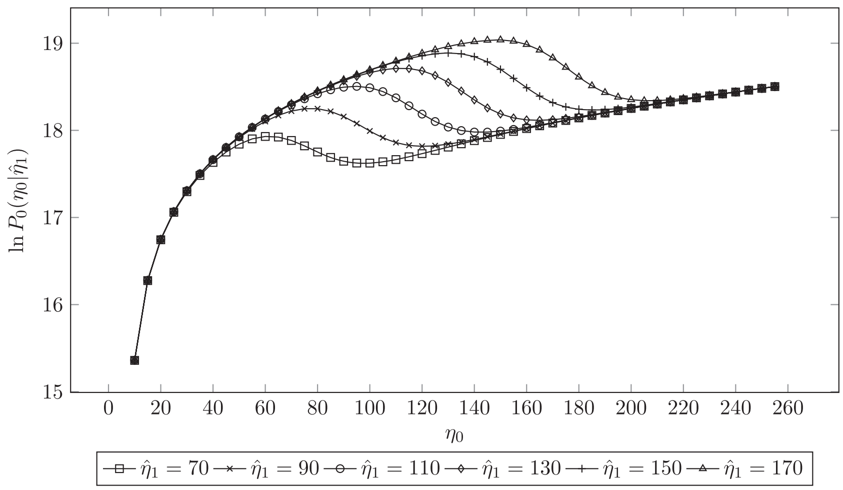


Figure 5. The curves of $\ln P_0(\eta_0|\hat{\eta}_1)$ with different $\hat{\eta}_1$ under the Case (a) of Proposition 3.8 in the two-unit scenario.

the only STU can also be seen as the representative of the entire STU industry, this sensitivity analysis can still help shape regulatory policies targeting at adjusting and meeting the desired market share of not only a single STU, but also the entire STU industry including multiple homogenous units, which in turn helps to achieve environmental sustainability targets.

3.3.1 *Influence of the STU's gate fee*

From Proposition 3.4, if $U(\tilde{\eta_0}|\boldsymbol{\eta}_{0-}) \geqslant 0$, we always have $\eta_0^{\dagger} = \overline{\eta}_0$. The following gives the corresponding results in the case that $U(\tilde{\eta_0}|\boldsymbol{\eta}_{0-}) < 0$ (i.e. when the payoff function of GTU is not always non-decreasing as illustrated in Figure 4).

Proposition 3.8 *Assume a two-unit scenario with $U(\tilde{\eta_0}|\eta_1) < 0$. Given a certain STU's gate fee $\hat{\eta}_1$, let $\hat{\eta}_0^-$ be the GTU's gate fee corresponding to the local maximum obtained by solving the model (12). Then, (a) if $\hat{z}_0^-/\hat{z}_1 > \sqrt{\delta_1}$, where $\hat{z}_0^- = e^{\beta_0-\alpha_0\hat{\eta}_0^-}$ and $\hat{z}_1 = e^{\beta_1-\alpha_1\hat{\eta}_1}$, then η_0^- is positively correlated with the η_1 at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$; and (b) if $\hat{z}_0^-/\hat{z}_1 < \sqrt{\delta_1}$, then η_0^- is negatively correlated with the η_1 at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$.*

We first discuss Case (a). The condition $\hat{z}_0^-/\hat{z}_1 > \sqrt{\delta_1}$ has two implications: (1) the attractiveness of the GTU is comparable to that of the STU, i.e. the competition is near-homogeneous. Therefore, if the STU decreases its gate fee setting, the GTU's local maximum gate fee also decreases in response, to re-capture the market share loss. However, such price competition is not a long-term solution due to its harmfulness to the revenue stream of treatment units. (2) the residual fraction term $\sqrt{\delta_1}$ is small. If there is almost no residual from the STU for post-treatment, the GTU will need to compete exclusively for the input feedstock.

Additional information in Case (a) can be observed from Figure 5, which shows the log-payoff function $\ln P_0(\eta_0|\hat{\eta}_1)$ based on the data in the Example 1. Firstly, all the curves converge when η_0 is much lower or higher than $\hat{\eta}_1$. This is because when η_0 is much lower than $\hat{\eta}_1$, its market share $S_0(\eta_0|\hat{\eta}_1)$ will approach 100%. As a result, the value of $\ln P_0(\eta_0|\hat{\eta}_1)$ will approximately be $\ln \omega(\eta_0 + \mu_0)$, which is presented as the left bold line segment in the Figure 5. Secondly, when η_0 is higher than $\hat{\eta}_1$, the value of $S_0(\eta_0|\hat{\eta}_1)$ will approach 0%. Consequently, the value of $\ln P_0(\eta_0|\hat{\eta}_1)$ will approximately be $\ln \omega\delta_1(\eta_0 + \mu_0)$, which is presented as the right bold line segment in the Figure 5. In the latter situation, the GTU almost exclusively recovers profits from post-treatment of STUs' residuals, and almost do not treat input feedstock directly. Finally, it can be seen that given the η_0, the $\ln P_0(\eta_1|\eta_0)$ is increasing in η_1, which validates the results in Corollary 3.5.

Next, we consider Case (b) of Proposition 3.8, which assumes the condition that $\hat{z}_0^-/\hat{z}_1 < \sqrt{\delta_1}$, i.e. the GTU's local maximum gate fee $\hat{\eta}_0^-$ is significantly higher than the STU's gate fee $\hat{\eta}_1$. In Figure 6, all the $\hat{\eta}_1$ are very small, and the log-payoff $\ln P_0(\eta_0|\hat{\eta}_1)$ is relatively insensitive to changes in η_0 around the local maximum $\hat{\eta}_0^-$. Because of the comparatively low $\hat{\eta}_1$, the attractiveness of the GTU is insufficient for competition. On the other hand, in this case, the treatment of residuals from the STU forms a substantial revenue stream for the GTU. Therefore, the GTU tends to recover more revenues from post-treatment of the residuals by increasing his own gate fee and thus give up competing for the market share. This finding is interesting and helpful for the regulator who attempts to promote the utilisation of the STUs under competition.

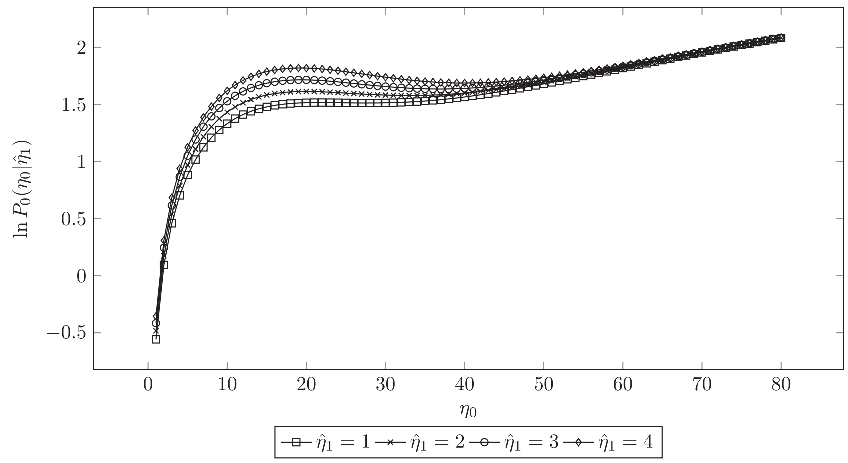


Figure 6. The curves of $\ln P_0(\eta_0|\hat{\eta}_1)$ with different $\hat{\eta}_1$ under the Case (b) of Proposition 3.8 in the two-unit scenario.

3.3.2 *Influence of the STU's operation profit rate and residual coefficient*

The operation profit rate μ_1 and residual coefficient δ_1 are two important parameters of the STU as they directly determine its profitability. In actual practice, μ_1 can be affected by economies of scale and various incentive policies from the regulator such as operation subsidy and tax rebates. The δ_1 can be influenced by the process efficiency improvements. A reduction of δ_1 in the STU's payoff function can also be due to a gate fee subsidy for residues post-treatment. These can then in turn influence the market share. For example, in the two-unit scenario, suppose that the STU selects the payoff maximising gate fee η_1^* over the domain $(\underline{\eta_1}, \infty)$, and the GTU selects the local maximum gate fee η_0^-. According to Corollary 3.2, increasing μ_1, or decreasing δ_1 will decrease the η_1^*. Besides, according to the Proposition 3.8, if the condition $z_0^-/z_1^* < \sqrt{\delta_1}$ is satisfied, a decrease in the η_1^* will lead to an increase in η_0^-, which corresponds to an increase in the STU's market share.

3.3.3 *Influence of the GTU's gate fee upper bound*

Price restriction is a direct approach for the regulator to control the waste treatment market. The aforementioned results are concerned with the effect of controlling the STU's gate fee for improving its utilisation (i.e. increasing market share). We now consider a situation where the market share can be controlled by the GTU's gate fee upper bound.

COROLLARY 3.9 *In the two-unit scenario, suppose both STU and GTU select their respective gate fee upper bounds $\overline{\eta}_1$ and $\overline{\eta}_0$ as their optimal equilibrium gate fees. The market share of STU will then increase if the $\overline{\eta}_0$ increases while satisfying either of the following cases: (a) $U(\tilde{\eta}_0|\overline{\eta}_1) \geqslant 0$; (b) $U(\tilde{\eta}_0|\overline{\eta}_1) < 0$ and $\overline{\eta}_0 \in (\underline{\eta_0}, \eta_0^-] \cup (\eta_0^\Gamma, \infty)$.*

Essentially, Corollary 3.9 considers the cases when the gate fee upper bound is imposed on non-decreasing segments of the GTU's payoff function, and such that choosing the gate fee upper bound will always be optimal. As a result, the STU's optimal gate fee cannot further increase, while that of the GTU increases, and it receives higher input feedstock. On the other hand, the overall profits of the GTU also increase.

Although the sensitivity analysis provides some suggestions on how the gate fee and market share of the treatment units can influence each other, it only gives a 'local' description of the system behaviour about some initial system state. To obtain the optimal equilibrium gate fee and market share outcomes, the sensitivity analysis can be performed repeatedly, or using an numerical algorithm such as that in the Appendix 2. We will present further computational studies in Section 5.

4. A central-control model

In this section, we consider the scenario where the local regulator takes the role of determining the gate fees for all treatment units in the system considered. The advantage of the central-control policy is that the regulated environmental and economic feasibility requirements can be fulfilled more directly. Based on the treatment unit economic models, a central-control model is proposed as follows for comparing the market competition and central-control method in the system considered.

$$\max_{\boldsymbol{\eta}} \quad \min\{P_f(\boldsymbol{\eta}),\ \forall f \in \mathcal{I}\} \tag{14}$$

$$\text{s.t.} \quad (3),\ (4),\ (6),\ (7) \text{ and}$$

$$S_f(\boldsymbol{\eta}) \geqslant \underline{S_f},\ \forall f \in \mathcal{I} \tag{15}$$

$$\omega \left[S_0(\boldsymbol{\eta}) + \sum_{f \in \mathcal{I}} \underline{S_f} \delta_f \right] (\eta_0 + \mu_0) \geqslant \underline{P_0} \tag{16}$$

In the above, (14) represents the objective function of maximising the minimal payoff across all STUs. (15) ensures the market share of each STU should be at least above a desired lower bound $\underline{S_f}$, i.e. the environmental sustainability targets. And (16) implies that the payoff of GTU should exceed a desired lower bound $\underline{P_0}$ when the market share of each STU is at its desired level. Its application is to secure the economic feasibility of GTU.

To solve the central-control model, we discuss the property of $S_f(\boldsymbol{\eta})$ and $P_f(\boldsymbol{\eta})$.

PROPOSITION 4.1 *For every $f \in \mathcal{I}$, the market share function $S_f(\boldsymbol{\eta})$ and payoff function $P_f(\boldsymbol{\eta})$ are log-concave with respect to the gate fee vector $\boldsymbol{\eta}$ when $\eta_f > \underline{\eta_f}$.*

Based on the Proposition 4.1, we replace the payoff function $P_f(\boldsymbol{\eta})$ in the (14) with its logarithmic form $\ln P_f(\boldsymbol{\eta})$, which does not change the optimal solution. Besides, we denote $p = \min\{\ln P_f(\boldsymbol{\eta}),\ \forall f \in \mathcal{I}\}$, and take logarithm for both sides of (15). Then the central-control model above can be equivalently reformulated as follows.

$$\max_{\boldsymbol{\eta}} \quad p \tag{17}$$

$$\text{s.t.} \quad (3),\ (4),\ (6),\ (7) \text{ and}$$

$$\ln S_f(\boldsymbol{\eta}) \geqslant \ln \underline{S_f},\ \forall f \in \mathcal{I} \tag{18}$$

$$\ln P_f(\boldsymbol{\eta}) \geqslant p,\ \forall f \in \mathcal{I} \tag{19}$$

$$S_0(\boldsymbol{\eta}) \geqslant \frac{1}{\omega} \left[\frac{\underline{P_0}}{(\eta_0 + \mu_0)} - \sum_{f \in \mathcal{I}} \underline{S_f} \delta_f \right] \tag{20}$$

It can be seen that the reformulated central-control model above is a convex optimisation problem that can be solved by off-the-shelf solvers. Besides, (20) implies that imposing payoff lower bound for the GTU in (16) essentially ensures it can at least achieve a market share lower bound $\left[\underline{P_0}/(\eta_0 + \mu_0) - \sum_{f \in \mathcal{I}} \underline{S_f} \delta_f\right]/\omega$. Therefore, the proposed central-control model optimises the economic performance of STU with guaranteeing every unit to have a certain amount of market share in the system considered.

5. Computational studies

In this section, we perform numerical studies of the proposed symbiotic waste management system based on the case of Singapore, a land-scarce city state which has long faced an uphill struggle to recycle its food waste. Traditionally, incineration (i.e. GTU) is the only option for Singapore to treat its food waste with low energy recovery efficiency due to high moisture content. In 2008, the local environmental regulator implemented the anaerobic digestion (i.e. STU) for recycling the food waste with better energy recovery efficiency, less hazardous emission and landfill usage. However, the under-utilisation issue led to the shutdown of the largest private anaerobic digestion company IUT Global in three years since its inception (The Straits Times 2011). Consequently, the food waste recycling rate was only 13% in 2015, which is much lower than the overall MSW recycling rate of 61% in this city (NEA 2015). Therefore, a computational study providing decision support for sustaining the operation of a STU like anaerobic digestion in a competitive gate fee charge environment is of significant importance for the success of improving environmental sustainability in Singapore.

The computational study is conducted for a two-unit scenario, i.e. with one STU and one GTU. Based on the description in Section 2.2, Algorithm 1 in the Appendix 2 is proposed to simulate the competitive gate fee charge environment. The initial gate fee of STU and GTU is set to 100 SGD/ton and 80 SGD/ton, respectively. The initial gate fee lower bound of STU and GTU is set to 78.43 SGD/ton and 6.67 SGD/ton, respectively, based on their definitions in the Section 3 and the data setting in the Example 1. In contrast to the gate fee lower bound data, no prior study or report has published gate fee upper bound data in practical implementations in Singapore. Considering that the garbage collection fee was raised by around 20% in Singapore due to higher operating costs (The Straits Times 2013), we assume that the initial gate fee upper bound of STU and GTU is 120 SGD/ton and 100 SGD/ton, respectively, which are 20 SGD/ton higher than their initial gate fee settings.

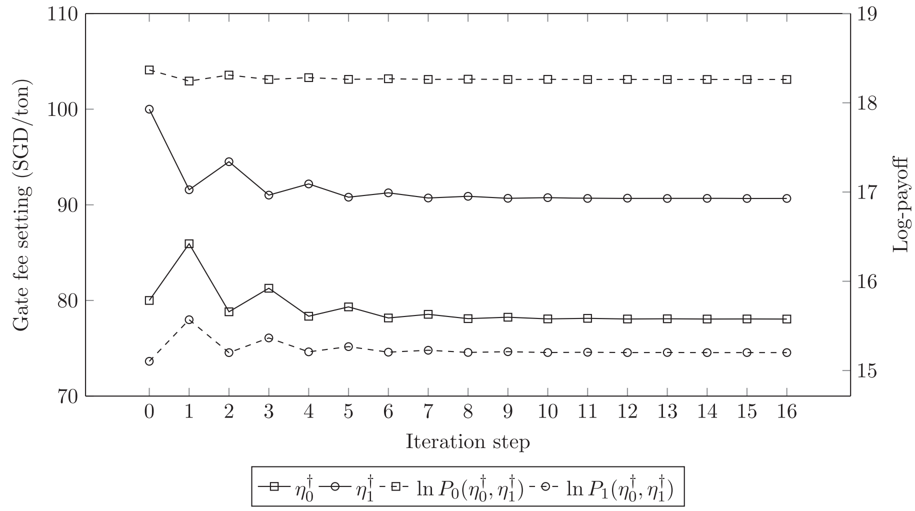


Figure 7. The gate fee setting and log-payoff of STU and GTU during competition in the basic simulation of two-unit scenario.

Besides, our extensive model testing also justifies that both units will not choose a gate fee that is higher than this initial gate fee upper bound setting as optimum during their competition. Therefore, we do not consider a higher gate fee upper bound setting in our computational studies. Other data setting are derived from the Example 1. We note that the main purpose of the computational studies is not to simulate the actual scenario in the Singapore context or predict the future system performance precisely, but rather to see the overall system's dynamic behaviour during market competition and validate the derived insights from model analysis. Besides, to mitigate the deviation from data assumptions, we will also perform some simulations to discuss how can these assumed data settings influence the competitive behaviour and performance of STU and GTU during their interactions. Finally, the stopping criteria ξ for judging the equilibrium state in the Algorithm 1 is set to 0.01 SGD/ton.

5.1 *Numerical simulation and analysis of waste treatment operators*

We first present the basic simulation results of the two-unit scenario based on the initial data setting. Figure 7 illustrates the gate fee and log-payoff of STU and GTU in each iteration step until the system reaches equilibrium. The time interval between two adjacent iteration steps can be regarded as the time taken for market to make response to the units' gate fee adjustment. It can be seen that the gate fees of STU and GTU are smaller than their upper bounds at every iteration step, which means that the STU chooses the payoff maximising gate fee η_1^* over the domain $(\underline{\eta_1}, \infty)$ and the GTU chooses the local maximum gate fee η_0^- as their optimum, respectively. Moreover, at every iteration, both units follow their opponent's gate fee adjustment direction at the previous iteration step. Therefore, the 'round-by-round' interactions and behaviours of these two units validate the results of Corollary 3.2 and Case (a) of Proposition 3.8. It implies that the system is operating in the market competition regime. The equilibrium gate fee of STU and GTU is 90.67 SGD/ton and 78.06 SGD/ton, respectively, and the market share of STU is only 22.09%. In addition, the log-payoff of STU is 15.2 (approximate to 3,992,787 SGD), which is lower than 18.16 (approximate to 77,052,688 SGD) of the GTU. These simulation results are qualitatively similar to the current Singapore situation, where new technology waste treatment remains a very niche market. They also validate that the proposed model can effectively reflect the symbiotic system structure and gate fee competition process with private sector participation. Based on some of the findings in Section **??**, in the following, numerical simulations are performed by changing different model parameters to evaluate the effectiveness of different intervention approaches to improve the market share of the STU.

5.1.1 *STU gate fee upper bound*

The computational study on the influence of STU's gate fee upper bound $\overline{\eta}_1$ in the two-unit scenario is presented in this section. Specifically, we reduce $\overline{\eta}_1$ from 120 SGD/ton to 80 SGD/ton and perform the numerical simulation. Table 2 shows the optimal equilibrium states of the two units under different $\overline{\eta}_1$. It can be seen that the equilibrium gate fees and log-payoffs

Table 2. Performance comparison of STU and GTU at the optimal equilibrium states with different $\overline{\eta}_1$ in the two-unit scenario.

		$\overline{\eta}_1$ (SGD/ton)					
Parameter	Unit	120	90	87.5	85	82.5	80
η_0^∞	SGD/ton	78.06	77.51	75.43	73.38	71.35	69.33
η_1^∞	SGD/ton	90.67	90	87.5	85	82.5	80
$S_0(\eta_0^\infty, \eta_1^\infty)$	%	77.91	77.72	76.97	76.17	75.31	74.4
$S_1(\eta_0^\infty, \eta_1^\infty)$	%	22.09	22.28	23.03	23.83	24.69	25.6
$\ln P_0(\eta_0^\infty, \eta_1^\infty)$	–	18.16	18.25	18.22	18.18	18.14	18.1
$\ln P_1(\eta_0^\infty, \eta_1^\infty)$	–	15.2	15.17	15.04	14.88	14.67	14.38

Table 3. Performance comparison of STU and GTU at the optimal equilibrium states with different d_1 in the two-unit scenario.

		d_1					
Parameter	Unit	1	0.9	0.7	0.5	0.3	0.1
η_0^∞	SGD/ton	78.06	74.56	68.39	63.12	58.58	54.63
η_1^∞	SGD/ton	90.67	87.24	81.24	76.14	71.76	67.96
$S_0(\eta_0^\infty, \eta_1^\infty)$	%	77.91	78.05	78.33	78.61	78.88	79.13
$S_1(\eta_0^\infty, \eta_1^\infty)$	%	22.09	21.95	21.67	21.39	21.12	20.87
$\ln P_0(\eta_0^\infty, \eta_1^\infty)$	–	18.16	18.2	18.09	17.99	17.89	17.79
$\ln P_1(\eta_0^\infty, \eta_1^\infty)$	–	15.2	15.19	15.18	15.16	15.14	15.13

of both units decrease when the STU gate fee upper bound decreases, due to market competition effects. These results also validate the behaviour described in the Case (a) of Proposition 3.8, where the GTU is operating in the market competition regime for feedstock. It indicates that the policy of reducing the $\overline{\eta}_1$ can help to ease the financial burden of treatment service users. However, the equilibrium market share of STU when $\overline{\eta}_1 = 80$ SGD/ton is only 3.51% higher than that when $\overline{\eta}_1 = 120$ SGD/ton. This suggests that depressing $\overline{\eta}_1$ is not very effective in improving the STU's utilisation. Furthermore, decreasing $\overline{\eta}_1$ can cause significant payoff slump of the STU, which indicates that this policy can adversely affect its economic feasibility.

5.1.2 *Residual post-treatment discount*

We assume the scenario where the GTU can provide STU a gate fee discount for residuals post-treatment. Denote the residual post-treatment discount factor as d_f for $f \in \mathcal{I}$, where $d_f \in [0, 1]$. The STU payoff function (8) is reformulated as

$$P_f(\boldsymbol{\eta}) = \omega S_f(\boldsymbol{\eta}) \left(\eta_f + \mu_f - \delta_f d_f \eta_0\right) \tag{21}$$

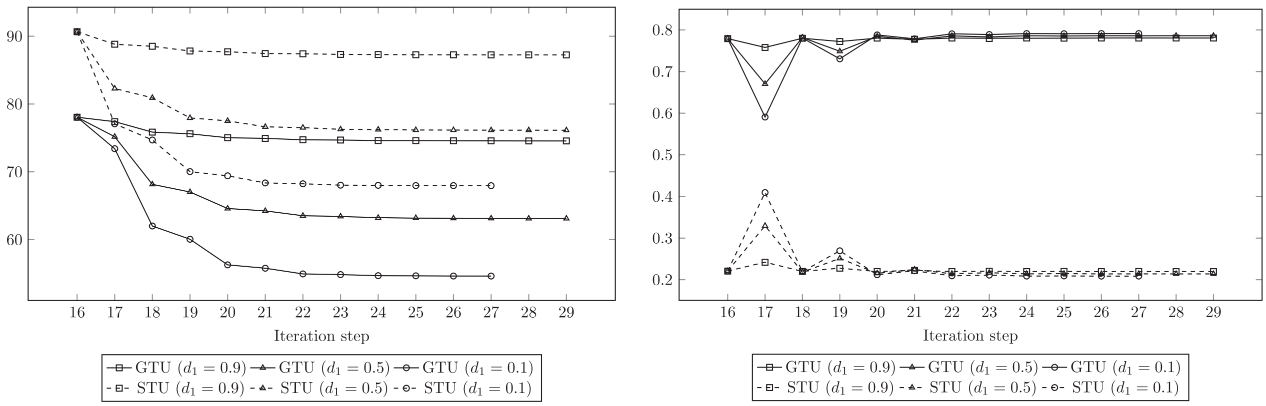
for all $f \in \mathcal{I}$, and the GTU payoff function (9) is reformulated as

$$P_0(\boldsymbol{\eta}) = \omega S_0(\boldsymbol{\eta})(\eta_0 + \mu_0) + \sum_{f \in \mathcal{I}} \omega S_f(\boldsymbol{\eta})\, \delta_f \left(d_f \eta_0 + \mu_0\right). \tag{22}$$

Note that the original payoff formulations (8) and (9) without residual post-treatment discount are the special cases of the above formulations when $d_f = 1$ for all $f \in \mathcal{I}$.

Table 3 shows the performance comparison of the two units at different d_1. It can be seen that the gate fees and log-payoffs of both units decrease as larger gate fee discounts for the STU are given. This suggests that the residue post-treatment discount from GTU actually benefits the refuse collectors, rather than the STU. In addition, it can be observed that decreasing d_1 in fact reduces (increases) the STU's (GTU's) market share slightly, which is opposite of the intended purpose of increasing the STU utilisation.

Figures 8(a) and (b) illustrate the gate fee and market share adjustment process after the d_1 is reduced from 1.0 to 0.9, 0.5 and 0.1 separately. From Figure 8(a), it can be seen that the STU gate fee first decreases when the discount is introduced. This is because the discount factor d_1 leads to a lower gate fee break-even point (i.e. gate fee lower bound) $-\eta_1 - \mu_1 + \delta_1 d_1 \eta_0$, which in turn decreases the payoff maximising gate fee η_1^* over the domain $(\underline{\eta}_1, \infty)$, which validates the Corollary 3.2. In response, the GTU reduces its gate fee due to the market competition, which is further reinforced by the fact that the residuals



(a) Gate fee setting (SGD/ton) (b) Market share

Figure 8. The gate fee setting and market share adjustment process of STU and GTU after the reduction of d_1 in the two-unit scenario.

Table 4. Performance comparison of STU and GTU at the optimal equilibrium states with different μ_1 in the two-unit scenario.

		μ_1 (SGD/ton)					
Parameter	Unit	−53.63	−48.63	−43.63	−38.63	−33.63	−28.63
η_0^∞	SGD/ton	78.06	72.89	67.94	63.26	58.9	54.91
η_1^∞	SGD/ton	90.67	84.39	78.26	72.29	66.55	61.06
$S_0(\eta_0^\infty, \eta_1^\infty)$	%	77.91	75.96	73.73	71.16	68.25	64.91
$S_1(\eta_0^\infty, \eta_1^\infty)$	%	22.09	24.04	26.27	28.84	31.75	35.09
$\ln P_0(\eta_0^\infty, \eta_1^\infty)$	–	18.16	18.17	18.07	17.97	17.87	17.76
$\ln P_1(\eta_0^\infty, \eta_1^\infty)$	–	15.2	15.31	15.43	15.56	15.7	15.85

treatment market is now less attractive due to the discount. Furthermore, from Figure 8(b), the reduction of d_1 only improves the utilisation of STU in the short-term. In fact, regardless of the amount of d_1 enforced, the market shares return to the original values before the implementation of the discount.

5.1.3 *STU operation profit rate*

We now vary the STU operation profit rate μ_1 in the simulation. Recall that μ_1 is the recovered product sales per unit less than all cost components except the residual post-treatment service gate fee. Table 4 presents the results of the two-unit system under different values of μ_1. It can be seen that the gate fees of both units decrease with increasing μ_1, and the log-payoff of STU increases. These simulation results validate the behaviour described in Corollary 3.2, which says that the optimal gate fee of the STU should decrease as operations profit rate μ_1 increases, and the Case (a) of the Proposition 3.8, which describes the market competitive nature of the GTU. In summary these results indicate that increasing μ_1 has the effect of reallocating benefits from the GTU to the STU and service users. Furthermore, the market share of STU increases with increasing μ_1, implying that it is an effective approach to improve its utilisation. However, Table 4 also shows that increasing μ_1 erodes the profitability of GTU. Hence, the policy of increasing the μ_1 should be carefully implemented to balance the economic considerations of the GTU.

5.1.4 *GTU gate fee upper bound*

The Corollary 3.9 in the Section 3.3 has proposed several cases in which the market share of STU can be enhanced by increasing the gate fee upper bound of GTU, $\overline{\eta}_0$. We now simulate the influence $\overline{\eta}_0$ on the system in these proposed cases. Here, the initial $\overline{\eta}_0$ and $\overline{\eta}_1$ are set to 65 SGD/ton and 85 SGD/ton, respectively. This can be seen as a form of stringent control to safeguard the welfare of the treatment service users. Consequently, every unit chooses its gate fee upper bound as optimum, which satisfies the prerequisite in Corollary 3.9. Table 5 presents the results when $\overline{\eta}_0$ varies in the range $(\underline{\eta_0}, \eta_0^-]$.

Table 5. Performance comparison of STU and GTU at the optimal equilibrium states when the $\overline{\eta}_0$ varies within the range $(\underline{\eta_0}, \eta_0^-]$ in the two-unit scenario.

Parameter	Unit	$\overline{\eta}_0$ (SGD/ton)			
		65	67	69	71
η_0^∞	SGD/ton	65	67	69	71
η_1^∞	SGD/ton	85	85	85	85
$S_0(\eta_0^\infty, \eta_1^\infty)$	%	88.08	85.81	83.2	80.22
$S_1(\eta_0^\infty, \eta_1^\infty)$	%	11.92	14.19	16.8	19.78
$\ln P_0(\eta_0^\infty, \eta_1^\infty)$	–	18.14	18.16	18.17	18.18
$\ln P_1(\eta_0^\infty, \eta_1^\infty)$	–	14.45	14.56	14.68	14.78

Table 6. Simulation results of central-control model with different $\underline{S_1}$ in the two-unit scenario.

Parameter	Unit	$\underline{S_1}$ (%)						
		0	10	20	30	40	50	60
$\eta_0^\dagger$	SGD/ton	100	100	100	100	100	100	100
$\eta_1^\dagger$	SGD/ton	102.45	102.45	102.45	102.45	102.45	100	95.95
$S_0(\eta_0^\dagger, \eta_1^\dagger)$	%	56.1	56.1	56.1	56.1	56.1	50	40
$S_1(\eta_0^\dagger, \eta_1^\dagger)$	%	43.9	43.9	43.9	43.9	43.9	50	60
$\ln P_0(\eta_0^\dagger, \eta_1^\dagger)$	–	18.33	18.33	18.33	18.33	18.33	18.27	18.16
$\ln P_1(\eta_0^\dagger, \eta_1^\dagger)$	–	16.22	16.22	16.22	16.22	16.22	16.20	16.07
p	–	16.22	16.22	16.22	16.22	16.22	16.20	16.07

It can be seen that both units set their optimum gate fees at their upper bounds as $\overline{\eta}_0$ varies over $(\underline{\eta_0}, \eta_0^-]$. These simulation results validate the results of Proposition 3.1, Corollary 3.2 and Proposition 3.4. Also, increasing $\overline{\eta}_0$ leads to the market share growth of STU, which validates the Case (b) of Corollary 3.9. Finally, the increase in the optimal gate fees and log-payoffs of both units with increasing $\overline{\eta}_0$ implies that the interests are reallocated from treatment users to operators. This is a trade-off that policy-makers should take note of.

5.2 *Performance comparison with central-control model*

We next compare the computational results of the market competition and central-control model (14)–(16) in the two-unit scenario. The data setting in the Example 1 is adopted. In the central-control model, the GTU's desired log-payoff $\ln \underline{P_0}$ in constraint (16) is set to 18.16, which is the initial equilibrium value achieved in Section 5.1 (see Figure 7). Table 6 presents the results of the central-control model under different desired market share of STU $\underline{S_1}$. To ensure that achieving the $\ln \underline{P_0}$ is feasible, $\underline{S_1}$ can only be increased to 60%. First, it can be seen that the results are the same when $\underline{S_1}$ is between 0% and 40%, since the optimal market share $S_1(\eta_0^\dagger, \eta_1^\dagger)$ is larger than $\underline{S_1}$. When $\underline{S_1}$ is set to above 43.9%, the central-control attempts to achieving the market share target by decreasing the $\eta_1^\dagger$ while holding $\eta_0^\dagger$ unchanged. As $\underline{S_1}$ increases, both $\ln P_0(\eta_0^\dagger, \eta_1^\dagger)$ and $\ln P_1(\eta_0^\dagger, \eta_1^\dagger)$ decrease, implying that the improvement in the system's environmental performance is at the expense of its economic performance. Finally, the central-control model can achieve $S_1(\eta_0^\dagger, \eta_1^\dagger)$ up to 60% while simultaneously satisfying $\ln \underline{P_0}$, and achieving a higher payoff for the STU (i.e. $\ln P_1(\eta_0^\dagger, \eta_1^\dagger) = 16.07$) compared to that in the market competition model (i.e. $\ln P_1(\eta_0^\dagger, \eta_1^\dagger) = 15.2$, see Section 5.1). However, the optimal gate fees $\eta_0^\dagger$ and $\eta_1^\dagger$ achieved in the central-control model are much higher than those in the market competition model. Therefore, the central-control method has a more effective system performance from the environmental perspective, and also from the perspective of treatment operator benefits. While the market competition model results in lower costs for treatment users . This is a trade-off that policy-makers should balance carefully.

5.3 *Results and discussion*

In summary, compared to the prior waste management system models, the above simulation results exhibit that our proposed model can help decision-makers better understand the operation status of different private waste treatment unit operators during market competition. And it can further provide decision-makers with managerial insights into making effective policies for leading the system to achieve the expected environmental sustainability target. In particular, according to the simulation result in the Singapore context, the STU is at a competitive disadvantage compared to the GTU in the symbiotic waste management system. It indicates that without appropriate policy interventions from the local regulator, the new private STU operators may not be able to sustain during the market competition. The simulation on the impact of several system parameters demonstrates that, the policy of increasing the STU's operation profit rate is more effective than depressing its gate fee upper bound for promoting the utilisation of STU, so as to improve the system's environmental sustainability performance at the equilibrium state. However, these two policies can adversely affect the profitability of STU and GTU. Besides, the policy of decreasing the GTU's gate fee upper bound can also help enhance the utilisation of STU at the equilibrium state when both units select their gate fee upper bounds. But the service users have to pay more for the STU and GTU service delivery. Moreover, in contrast to the intended purpose of increasing the STU's utilisation, the policy of imposing a residual post-treatment discount reduces the utilisation of STU and deteriorates the system's economic performance at the equilibrium state. Finally, the comparison of simulation results between the market competition and central-control model demonstrates that, the central-control method performs better in terms of system's economic and environmental impact, while the market competition method favours more of alleviating the service users' financial burden.

6. Conclusion

In this paper, a decision support model is developed for optimising the payoff of self-interested specialised and GTU (STU and GTU) operators in a symbiotic MSW treatment system under market competition. All operators in the MSW treatment market compete for input MSW by setting their preferred gate fee charge levels. A comprehensive analysis was conducted to discuss the properties of different unit operators' payoff functions, and to determine their optimal gate fee strategies. Moreover, a sensitivity analysis on several key parameters was presented to discuss the 'one-round' interaction between different unit operators in a reduced two-unit scenario. It showed that by adjusting the STU's gate fee, operation profit rate and residue coefficient, and the GTU's gate fee upper bound, the utilisation of environmental-friendly STU can be promoted at some specific cases. In practical cases, this sensitivity analysis can help decision-makers to implement appropriate policy interventions for mitigating environmental sustainability issues. For comparison purposes, a central-control model was also established to maximise the minimal payoff across all STUs with simultaneously securing a payoff lower bound for GTU.

Using a case based on the Singapore waste management context, a computational study in the two-unit scenario was conducted by applying the proposed iterative algorithm to solve the decision support model. The simulation result revealed that the proposed model in this study can help decision-makers understand the economic performance of different private treatment unit operators during market competition in real cases. Besides, it showed that the proposed model can also facilitate the decision-makers to estimate the potential impact of different intervention policies, and to help understand the trade-off between different policies for achieving sustainability targets. For example in the Singapore case, although increasing the STU's operation profit rate and the GTU's gate fee upper bound under some specific conditions can help improve system's environmental sustainability performance at the optimal equilibrium state, it may deteriorate the system's economic performance. Finally, the comparison between the market competition and central-control method demonstrated that this study can help decision-makers clarify the gains and losses of implementing different waste management approaches for the symbiotic waste management system. In summary, we believe that these research outcomes can help tackle the under-utilisation issue of private innovative STU (e.g. IUT Global) to establish a sustainable symbiotic waste management system with private sector participation in Singapore.

In this work, the waste transferring facility in the MSW management system is not directly considered since we focus on the waste treatment process. If necessary, it can be included in an extended version of the proposed model by three possible ways: (1) The waste transferring facility can be regarded as a self-interested agent interacting with both waste collectors and treatment unit operators. In this way, a unique payoff function will be formulated for each waste transferring facility operator based on his available competitive strategy; (2) The waste transferring facility can be considered as a part of waste treatment unit to facilitate its transportation of waste received from waste collectors. By doing this, the location of waste transferring facility will be modelled as decision variables in the payoff function of waste treatment unit operator; (3) The waste treatment facility can be deemed as the property of local regulator. Then, the local regulator will take charge of designing waste transferring system to facilitate the transportation of collected wastes between waste collectors and treatment unit operators.

We also consider several other directions for future research. Firstly, more competitive strategies like service quality or route can be considered by the utility-based market share model in the proposed payoff formulation of waste treatment units.

Secondly, a more expensive, high fidelity simulation model can be developed and calibrated by using accurate customer preference data to obtain precise quantitative analysis of the interactions among multiple treatment unit operators. Also, the agent-based modelling approach can be applied to improve the proposed model by yielding more managerial insights in a multiple-unit scenario of the symbiotic waste management system. Lastly, the social aspects can be incorporated into the proposed model as a future extension.

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Appendix 1. Proofs

Proof of Proposition 3.1 For every $f \in \mathcal{I}$, when $\eta_f > \underline{\eta}_f$, its log-payoff function is

$$\ln P_f(\eta_f|\boldsymbol{\eta}_{f-}) = \ln \omega + \ln S_f(\eta_f|\boldsymbol{\eta}_{f-}) + \ln(\eta_f - \underline{\eta}_f). \tag{A1}$$

It can be seen that the first term of the above function is constant, and the third term is concave with respect to η_f. Therefore, we only need to prove the concavity of the log-function $\ln S_f(\eta_f|\boldsymbol{\eta}_{f-})$ with respect to η_f. Here, we have

$$\ln S_f(\eta_f|\boldsymbol{\eta}_{f-}) = \beta_f - \alpha_f \eta_f - \ln \sum_{f \in \mathcal{F}} z_f. \tag{A2}$$

Then, its first-order partial derivative with respect to η_f is

$$\frac{\partial \ln S_f(\eta_f|\boldsymbol{\eta}_{f-})}{\partial \eta_f} = -\alpha_f + \frac{\alpha_f z_f}{\sum_{f \in \mathcal{F}} z_f}, \tag{A3}$$

and its second-order partial derivative is

$$\frac{\partial^2 \ln S_f(\eta_f|\boldsymbol{\eta}_{f-})}{\partial \eta_f^2} = \frac{(\alpha_f z_f)^2 - \alpha_f^2 z_f \sum_{f \in \mathcal{F}} z_f}{(\sum_{f \in \mathcal{F}} z_f)^2} = \frac{\alpha_f^2 z_f (z_f - \sum_{f \in \mathcal{F}} z_f)}{(\sum_{f \in \mathcal{F}} z_f)^2} \leqslant 0.$$

Therefore, the log-function $\ln S_f(\eta_f|\boldsymbol{\eta}_{f-})$ is concave with respect to η_f when $\eta_f > \underline{\eta}_f, \forall f \in \mathcal{I}$. Since addition preserves concavity, the log-payoff function $\ln P_f(\eta_f|\boldsymbol{\eta}_{f-})$ is concave with respect to η_f when $\eta_f > \underline{\eta}_f, \forall f \in \mathcal{I}$.

Based on the concavity property, to determine the payoff maximising gate fee η_f^* over the domain $(\underline{\eta}_f, \infty)$, we consider the first-order derivative of log-payoff function $\ln P_f(\eta_f|\boldsymbol{\eta}_{f-})$ with respect to η_f, namely

$$\frac{\partial \ln P_f(\eta_f|\boldsymbol{\eta}_{f-})}{\partial \eta_f} = \frac{1}{\eta_f - \underline{\eta}_f} - \frac{\alpha_f \sum_{f' \in \mathcal{F}/\{f\}} z_{f'}}{\sum_{f \in \mathcal{F}} z_f}. \tag{A4}$$

It can be seen that if $\eta_f \to \underline{\eta_f}$, the value of $\partial \ln P_f(\eta_f|\boldsymbol{\eta}_{f-})/\partial\eta_f$ will approach positive infinity. Besides, if $\eta_f \to \infty$, the value of z_f will approach 0. Therefore, the value of $\partial \ln P_f(\eta_f|\boldsymbol{\eta}_{f-})/\partial\eta_f$ will approach $-\alpha_f$, which is less than 0. It means that $\ln P_f(\eta_f|\boldsymbol{\eta}_{f-})$ is neither a monotonically increasing or decreasing concave function when $\eta_f > \underline{\eta_f}$. Hence, η_f^* must satisfy the condition $\partial \ln P_f(\eta_f^*|\boldsymbol{\eta}_{f-})/\partial\eta_f = 0$, namely

$$\eta_f^* - \underline{\eta_f} - \frac{z_f^* + \sum_{f'\in\mathcal{F}/\{f\}} z_{f'}}{\alpha_f \sum_{f'\in\mathcal{F}/\{f\}} z_{f'}} = 0, \tag{A5}$$

where $z_f^* = e^{\beta_f - \alpha_f \eta_f^*}$.

Proof of Corollary 3.2 For every $f \in \mathcal{I}$, we reformulate the Equation (10) as

$$\left[\alpha_f(\eta_f^* - \underline{\eta_f}) - 1\right] \sum_{f'\in\mathcal{F}/\{f\}} z_{f'} - z_f^* = 0. \tag{A6}$$

For any $f' \in \mathcal{I}/\{f\}$, when the $\eta_{f'}$ increases (decreases), the $\sum_{f'\in\mathcal{F}/\{f\}} z_{f'}$ will decrease (increase). Therefore, the η_f^* should increase (decrease) to hold the Equation (A6). In addition, when the η_0 increases (decreases), the $\sum_{f'\in\mathcal{F}/\{f\}} z_{f'}$ will decrease (increase) and the $\underline{\eta_f}$ will increase (decrease). Therefore, the η_f^* should decrease (increase) as well to hold the Equation (A6). Besides, when the δ_f increases (decreases), the $\underline{\eta_f}$ will increase (decrease). Therefore, the η_f^* should increase (decrease) to hold the Equation (A6). Finally, when the μ_f increases (decreases), the $\underline{\eta_f}$ will decrease (increase). Therefore, the η_f^* should decrease (increase) to hold the Equation (A6).

Proof of Corollary 3.3 For the STU in the two-unit scenario, we have

$$\ln P_1(\eta_0, \eta_1^*) = \ln\omega + \beta_1 - \alpha_1\eta_1^* - \ln\left(z_0 + z_1^*\right) + \ln(\eta_1^* + \mu_1 - \delta_1\eta_0). \tag{A7}$$

As the payoff maximising gate fee η_1^* over the domain $(\underline{\eta_1}, \infty)$ is positively correlated with η_0 according to Corollary 3.2, we denote η_1^* as a function of η_0, namely $\eta_1^*(\eta_0)$. Therefore, we have $\partial\eta_1^*(\eta_0)/\partial\eta_0 > 0$. For convenience of representation, we denote $D = \partial\eta_1^*(\eta_0)/\partial\eta_0$. Then, we have

$$\frac{\partial \ln P_1(\eta_0, \eta_1^*(\eta_0))}{\partial\eta_0} = -\alpha_1 D + \frac{\alpha_0 z_0 + \alpha_1 z_1^* D}{z_0 + z_1^*} + \frac{D - \delta_1}{\eta_1^* + \mu_1 - \delta_1\eta_0}. \tag{A8}$$

According to the Equation (10), we can further obtain

$$\frac{\partial \ln P_1(\eta_0, \eta_1^*(\eta_0))}{\partial\eta_0} = -\alpha_1 D + \frac{\alpha_0 z_0 + \alpha_1 z_1^* D}{z_0 + z_1^*} + \frac{\alpha_1 z_0(D - \delta_1)}{z_0 + z_1^*} = \frac{\alpha_0 z_0 - \alpha_1\delta_1 z_0}{z_0 + z_1^*}. \tag{A9}$$

Therefore, if $\alpha_0 z_0 - \alpha_1\delta_1 z_0 > 0$, namely $\delta_1 < \alpha_0/\alpha_1$, the $\ln P_1(\eta_0, \eta_1^*)$ is positively correlated with η_0. And if $\delta_1 > \alpha_0/\alpha_1$, the $\ln P_1(\eta_0, \eta_1^*)$ is negatively correlated with η_0.

Proof of Proposition 3.4 When $\eta_0 \in (\underline{\eta_0}, \overline{\eta}_0]$, we have

$$\ln P_0(\eta_0|\boldsymbol{\eta}_{0-}) = \ln w + \ln\frac{z_0 + \sum_{f\in\mathcal{I}} z_f\delta_f}{\sum_{f\in\mathcal{F}} z_f} + \ln(\eta_0 - \underline{\eta_0}). \tag{A10}$$

Denoting that $\delta_0 = 1$ and $z_0 = z_0\delta_0$, we have

$$\ln P_0(\eta_0|\boldsymbol{\eta}_{0-}) = \ln w + \ln\frac{\sum_{f\in\mathcal{F}} z_f\delta_f}{\sum_{f\in\mathcal{F}} z_f} + \ln(\eta_0 - \underline{\eta_0}). \tag{A11}$$

Therefore, the first-order derivative of $\ln P_0(\eta_0|\boldsymbol{\eta}_{0-})$ with respect to η_0 is

$$\begin{aligned}\frac{\partial \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})}{\partial\eta_0} &= -\frac{\alpha_0\delta_0 z_0}{\sum_{f\in\mathcal{F}} z_f\delta_f} + \frac{\alpha_0 z_0}{\sum_{f\in\mathcal{F}} z_f} + \frac{1}{\eta_0 - \underline{\eta_0}}\\ &= \frac{\sum_{f\in\mathcal{F}} z_f\delta_f \sum_{f\in\mathcal{F}} z_f - \alpha_0 z_0(\eta_0 - \underline{\eta_0})(\sum_{f\in\mathcal{I}} z_f - \sum_{f\in\mathcal{I}} z_f\delta_f)}{(\eta_0 - \underline{\eta_0})\sum_{f\in\mathcal{F}} z_f\delta_f \sum_{f\in\mathcal{F}} z_f}.\end{aligned} \tag{A12}$$

It can be seen that the sign of the function $\partial \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0$ is only determined by its numerator as its denominator is definitely positive. From the definition of function $U(\boldsymbol{\eta})$, it is easy to know that: (a) when $U(\eta_0|\boldsymbol{\eta}_{0-}) > 0$, then $\partial \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0 > 0$; (b) when $U(\eta_0|\boldsymbol{\eta}_{0-}) = 0$, then $\partial \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0 = 0$; (c) when $U(\eta_0|\boldsymbol{\eta}_{0-}) < 0$, then $\partial \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0 < 0$. Furthermore, when $\eta_0 > \underline{\eta_0}$, the convexity of $U(\eta_0|\boldsymbol{\eta}_{0-})$ can be easily proved by applying the second-order condition and the convexity additivity property. Then we have

$$\frac{\partial U(\eta_0|\boldsymbol{\eta}_{0-})}{\partial\eta_0} = \frac{\alpha_0(\sum_{f\in\mathcal{I}} z_f \sum_{f\in\mathcal{F}} z_f\delta_f - z_0\sum_{f\in\mathcal{F}} z_f)}{\sum_{f\in\mathcal{F}} z_f \sum_{f\in\mathcal{F}} z_f\delta_f} - \frac{1}{\eta_0 - \underline{\eta_0}}. \tag{A13}$$

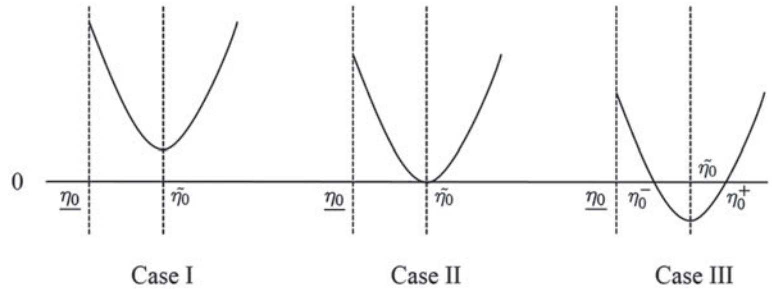

Figure A1. The three cases of the function $U(\eta_0|\boldsymbol{\eta}_{0-})$.

It can be seen that if $\eta_0 \to \underline{\eta_0}$, the value of $\partial U(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0$ will approach negative infinity. Besides, if $\eta_0 \to \infty$, the value of $\partial U(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0$ will approach α_0, which is greater than 0. Therefore, the $U(\eta_0|\boldsymbol{\eta}_{0-})$ is neither a monotonically increasing nor a monotonically decreasing convex function. And the value of $\tilde{\eta_0}$ is unique, and we must have

$$\tilde{\eta_0} - \underline{\eta_0} - \frac{\sum_{f\in\mathcal{F}} z_f \sum_{f\in\mathcal{F}} z_f \delta_f}{\alpha_0(\sum_{f\in\mathcal{I}} z_f \sum_{f\in\mathcal{F}} z_f \delta_f - z_0 \sum_{f\in\mathcal{F}} z_f)} = 0, \tag{A14}$$

where $\tilde{z}_0 = e^{\beta_0 - \alpha_0\tilde{\eta}_0}$. The value of $\tilde{\eta}_0$ can be directly solved by off-the-shelf solvers.

Based on the value of $\tilde{\eta_0}$, we discuss the three cases of the graph of $U(\eta_0|\boldsymbol{\eta}_{0-})$ when $\eta_0 > \underline{\eta_0}$, which are shown in the Figure A1. In the Case I, we have $U(\tilde{\eta_0}|\boldsymbol{\eta}_{0-}) > 0$ for all $\eta_0 \in (\underline{\eta_0}, \overline{\eta}_0]$, which indicates that the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is increasing in η_0. Therefore, the $\eta_0^{\dagger}$ should be set to $\overline{\eta}_0$. In the Case II, we have $U(\tilde{\eta_0}|\boldsymbol{\eta}_{0-}) = 0$, which indicates that the $U(\eta_0|\boldsymbol{\eta}_{0-}) \geqslant 0$ for all $\eta_0 \in (\underline{\eta_0}, \overline{\eta}_0]$. Therefore, the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is non-decreasing in η_0, and the $\eta_0^{\dagger}$ should be set to $\overline{\eta}_0$. In the Case III, we have $U(\tilde{\eta_0}|\boldsymbol{\eta}_{0-}) < 0$. Then, there must exists two intersection points (gate fees) that let $U(\eta_0|\boldsymbol{\eta}_{0-}) = 0$. Here, we denote these two gate fees as η_0^- and η_0^+, where $\eta_0^- < \eta_0^+$. Without considering the impact of $\overline{\eta}_0$ first, it can be seen that the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is decreasing in η_0 only when $\eta_0 \in (\eta_0^-, \eta_0^+)$, and increasing in η_0 when $\eta_0 \in (\underline{\eta_0}, \eta_0^-) \cup (\eta_0^+, \infty)$. Now we consider the impact of $\overline{\eta}_0$ on the choice of $\eta_0^{\dagger}$ in the Case III. If $\overline{\eta}_0 \in (\underline{\eta_0}, \eta_0^-]$, the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is increasing in η_0, which indicates that the $\eta_1^{\dagger}$ should be set to $\overline{\eta}_0$. The value of η_0^- can be calculated by the model (12). And since the function $U(\boldsymbol{\eta})$ is convex, the model (12) is a convex optimisation problem that can be solved by off-the-shelf solvers. If $\overline{\eta}_0 \in (\eta_0^-, \eta_0^+]$, the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is increasing in η_0 when $\eta_0 \in (\underline{\eta_0}, \eta_0^-]$, but decreasing in η_0 when $\eta_0 \in (\eta_0^-, \overline{\eta}_0]$. It indicates that the $\eta_0^{\dagger}$ should be set to η_0^-. If $\overline{\eta}_0 \in (\eta_0^+, \eta_0^{\Gamma})$, the ln $P_0(\eta_0|\boldsymbol{\eta}_{0-})$ is increasing in η_0 when $\eta_0 \in (\eta_0^+, \overline{\eta}_0]$. However, since the model (13) indicates that ln $P_0(\eta_0^{\Gamma}|\boldsymbol{\eta}_{0-}) = $ ln $P_0(\eta_0^-|\boldsymbol{\eta}_{0-})$, we have ln $P_0(\overline{\eta}_0|\boldsymbol{\eta}_{0-}) <$ ln $P_0(\eta_0^-|\boldsymbol{\eta}_{0-})$. Thus the $\eta_0^{\dagger}$ should be set to η_0^-. The value of η_0^{Γ} can be calculated by the model (13), which can be solved by numerical method. If $\overline{\eta}_0 = \eta_0^{\Gamma}$, we have ln $P_0(\overline{\eta}_0|\boldsymbol{\eta}_{0-}) = $ ln $P_0(\eta_0^{\Gamma}|\boldsymbol{\eta}_{0-}) = $ ln $P_0(\eta_0^-|\boldsymbol{\eta}_{0-})$, and the $\eta_0^{\dagger}$ can be set to either η_0^- or $\overline{\eta}_0$. If $\overline{\eta}_0 \in (\eta_0^{\Gamma}, \infty)$, we have ln $P_0(\overline{\eta}_0|\boldsymbol{\eta}_{0-}) >$ ln $P_0(\eta_0^-|\boldsymbol{\eta}_{0-})$. Thus the $\eta_0^{\dagger}$ should be set to $\overline{\eta}_0$.

Proof of Corollary 3.5 In the two-unit scenario, given the GTU's gate fee η_0, its log-payoff function is

$$\ln P_0(\eta_1|\eta_0) = \ln w + \ln \frac{z_0 + z_1\delta_1}{z_0 + z_1} + \ln(\eta_0 - \underline{\eta_0}). \tag{A15}$$

Therefore, we have

$$\frac{\partial \ln P_0(\eta_1|\eta_0)}{\partial \eta_1} = \frac{\alpha_1(1-\delta_1)z_0 z_1}{(z_0+z_1)(z_0+z_1\delta_1)}. \tag{A16}$$

Since $0 < \delta_1 < 1$, we definitely have $\partial \ln P_0(\eta_1|\eta_0)/\partial\eta_1 > 0$. Therefore, the ln $P_0(\eta_1|\eta_0)$ is increasing in the STU's gate fee η_1.

Proof of Corollary 3.6 According to the proof of Proposition 3.4, we have

$$\frac{\partial^2 \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})}{\partial \eta_0^2} = \frac{\alpha_0^2 z_0 \sum_{f\in\mathcal{I}} z_f \delta_f}{(\sum_{f\in\mathcal{F}} z_f\delta_f)^2} - \frac{\alpha_0^2 z_0 \sum_{f\in\mathcal{I}} z_f}{(\sum_{f\in\mathcal{F}} z_f)^2} - \frac{1}{(\eta_0 - \underline{\eta_0})^2}. \tag{A17}$$

It can be seen that the sufficient condition for letting the function $\partial^2 \ln P_0(\eta_0|\boldsymbol{\eta}_{0-})/\partial\eta_0^2$ be non-positive is

$$\frac{\alpha_0^2 z_0 \sum_{f\in\mathcal{I}} z_f \delta_f}{(\sum_{f\in\mathcal{F}} z_f\delta_f)^2} - \frac{\alpha_0^2 z_0 \sum_{f\in\mathcal{I}} z_f}{(\sum_{f\in\mathcal{F}} z_f)^2} \leqslant 0, \tag{A18}$$

which is equivalent to

$$\frac{\sum_{f\in\mathcal{F}} z_f}{\sum_{f\in\mathcal{F}} z_f\delta_f} \leqslant \sqrt{\frac{\sum_{f\in\mathcal{I}} z_f}{\sum_{f\in\mathcal{I}} z_f\delta_f}}. \tag{A19}$$

By denoting $r = \sqrt{\frac{\sum_{f\in\mathcal{I}} z_f}{\sum_{f\in\mathcal{I}} z_f\delta_f}}$, we have

$$z_0 \geqslant \frac{\sum_{f\in\mathcal{I}} z_f - r\sum_{f\in\mathcal{I}} z_f\delta_f}{r-1}. \tag{A20}$$

By taking natural logarithm for both sides, we have

$$\eta_0 \leqslant \frac{1}{\alpha_0}\left[\beta_0 - \ln\left(\sum_{f\in\mathcal{I}} z_f - r\sum_{f\in\mathcal{I}} z_f\delta_f\right) + \ln(r-1)\right]. \tag{A21}$$

Proof of Corollary 3.7 In the two-unit scenario, according to the definition of η_0^Γ in the Proposition 3.4, we have

$$\ln P_0(\eta_0^\Gamma|\boldsymbol{\eta}_{0-}) = \ln P_0(\eta_0^-|\boldsymbol{\eta}_{0-}), \tag{A22}$$

which is equivalent to

$$\frac{z_0^\Gamma + \sum_{f\in\mathcal{I}} z_f\delta_f}{z_0^\Gamma + \sum_{f\in\mathcal{I}} z_f}\cdot(\eta_0^\Gamma - \underline{\eta_0}) = \frac{z_0^- + \sum_{f\in\mathcal{I}} z_f\delta_f}{z_0^- + \sum_{f\in\mathcal{I}} z_f}\cdot(\eta_0^- - \underline{\eta_0}), \tag{A23}$$

where $z_0^\Gamma = e^{\beta_0-\alpha_0\eta_0^\Gamma}$ and $z_0^- = e^{\beta_0-\alpha_0\eta_0^-}$. Further, the Equation (A23) can be reformulated as

$$z_0^\Gamma z_1\left(\eta_0^\Gamma - \delta_1\eta_0^-\right) + (\eta_0^\Gamma - \eta_0^-)(z_0^\Gamma z_0^- + z_1^2\delta_1) + \underline{\eta_0}z_1(1-\delta_1)(z_0^- - z_0^\Gamma) = z_0^- z_1(\eta_0^- - \eta_0^\Gamma\delta_1). \tag{A24}$$

Since $\eta_0^\Gamma > \eta_0^-$ and $\delta_1 < 1$, the three terms in the LHS of Equation (A24) are positive. Therefore, the term in the RHS of Equation (A24) should be positive, which indicates that $\eta_0^\Gamma < \eta_0^-/\delta_1$.

Proof of Corollary 3.8 In the two-unit scenario, when $U(\tilde{\eta_0}|\eta_1) < 0$ always holds, according to the Proposition (3.4), we have

$$\begin{aligned} U(\eta_0,\eta_1) &= \ln(z_0+z_1) + \ln(z_0+z_1\delta_1) - \ln z_0 - \ln(\eta_0 - \underline{\eta_0}) - \ln\alpha_0 z_1(1-\delta_1) \\ &= \ln\left(\frac{z_0^2}{z_1} + z_1\delta_1 + z_0\delta_1 + z_0\right) - \ln z_0 - \ln(\eta_0-\underline{\eta_0}) - \ln\alpha_0(1-\delta_1). \end{aligned} \tag{A25}$$

We define $V(\eta_0,\eta_1) = z_0^2/z_1 + z_1\delta_1$. It is easy to know that $\partial V(\eta_0,\eta_1)/\partial\eta_1$ and $\partial U(\eta_0,\eta_1)/\partial\eta_1$ have the same sign. Then, we have

$$\frac{\partial V(\eta_0,\eta_1)}{\partial\eta_1} = z_1\alpha_1\left(\frac{z_0^2}{z_1^2} - \delta_1\right). \tag{A26}$$

Considering that the STU sets a certain gate fee $\hat{\eta}_1$, then we can calculate the corresponding local maximum gate fee of GTU $\hat{\eta}_0^-$ by substituting the $\hat{\eta}_1$ into the model (12). According to the definition of η_0^- and the Figure A1 in the Proposition 3.4, it is easy to know that

$$\frac{\partial U(\eta_0,\eta_1)}{\partial\eta_0}|_{\eta_0=\hat{\eta}_0^-,\eta_1=\hat{\eta}_1} < 0. \tag{A27}$$

Besides, according to the Equation (A26), if $\hat{z}_0^-/\hat{z}_1 > \sqrt{\delta_1}$ at the point $(\hat{\eta}_0^-,\hat{\eta}_1)$, we have

$$\frac{\partial U(\eta_0,\eta_1)}{\partial\eta_1}|_{\eta_0=\hat{\eta}_0^-,\eta_1=\hat{\eta}_1} > 0. \tag{A28}$$

And if $\hat{z}_0^-/\hat{z}_1 < \sqrt{\delta_1}$ at the point $(\hat{\eta}_0^-,\hat{\eta}_1)$, we have

$$\frac{\partial U(\eta_0,\eta_1)}{\partial\eta_1}|_{\eta_0=\hat{\eta}_0^-,\eta_1=\hat{\eta}_1} < 0. \tag{A29}$$

Now let us consider a change $\triangle\eta_1$ happening to the $\hat{\eta}_1$, and we denote that $\hat{\hat{\eta}}_1 = \hat{\eta}_1 + \triangle\eta_1$. Besides, we calculate the new corresponding local maximum gate fee of GTU $\hat{\hat{\eta}}_0^-$ by substituting the $\hat{\hat{\eta}}_1$ into the model (12), and denote that $\hat{\hat{\eta}}_0^- = \hat{\eta}_0^- + \triangle\eta_0$. Therefore, according to the proof of Proposition (3.4), we have

$$U(\hat{\hat{\eta}}_0^-,\hat{\hat{\eta}}_1) = U(\hat{\eta}_0^-,\hat{\eta}_1) = 0. \tag{A30}$$

It can be seen that, when the $\triangle\eta_1$ is small enough, we approximately have

$$U(\hat{\hat{\eta}}_0^-, \hat{\hat{\eta}}_1) = U(\hat{\eta}_0^-, \hat{\eta}_1) + \frac{\partial U(\eta_0, \eta_1)}{\partial \eta_0}|_{\eta_0=\hat{\eta}_0^-, \eta_1=\hat{\eta}_1} \cdot \triangle\eta_0 + \frac{\partial U(\eta_0, \eta_1)}{\partial \eta_1}|_{\eta_0=\hat{\eta}_0^-, \eta_1=\hat{\eta}_1} \cdot \triangle\eta_1. \tag{A31}$$

Based on the Equations (A30) and (A31), we can derive that

$$\frac{\partial U(\eta_0, \eta_1)}{\partial \eta_0}|_{\eta_0=\hat{\eta}_0^-, \eta_1=\hat{\eta}_1} \cdot \triangle\eta_0 + \frac{\partial U(\eta_0, \eta_1)}{\partial \eta_1}|_{\eta_0=\hat{\eta}_0^-, \eta_1=\hat{\eta}_1} \cdot \triangle\eta_1 = 0. \tag{A32}$$

Therefore, when $\hat{z}_0^-/\hat{z}_1 > \sqrt{\delta_1}$ at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$, if $\triangle\eta_1 > (<)0$, we should have $\triangle\eta_0 > (<)0$ to hold the Equation (A32). In other words, the η_0^- is positively correlated with the η_1 at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$. When $\hat{z}_0^-/\hat{z}_1 < \sqrt{\delta_1}$ at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$, if $\triangle\eta_1 > (<)0$, we should have $\triangle\eta_0 < (>)0$ to hold the (A32). In other words, the η_0^- is negatively correlated with η_1 at the point $(\hat{\eta}_0^-, \hat{\eta}_1)$.

Proof of Corollary 3.9 When the $\overline{\eta}_0$ increases while satisfying the Case (a) and (b), the GTU will always choose $\overline{\eta}_0$ as its optimal equilibrium gate fee according to the Proposition 3.4. Besides, according to the Proposition 3.1, the condition $\eta_1^\dagger = \overline{\eta}_1$ indicates that $\overline{\eta}_1 \leqslant \eta_1^*$. As the growth of $\overline{\eta}_0$ will increase η_1^* according to the analysis of Corollary 3.2, the optimal equilibrium gate fee of STU will always be $\overline{\eta}_1$ after the increase of $\overline{\eta}_0$. And since only the $\overline{\eta}_0$ increases, the STU's market share will increase.

Proof of Proposition 4.1 For every $f \in \mathcal{I}$, when $\eta_f > \underline{\eta_f}$, we have

$$\ln S_f(\boldsymbol{\eta}) = \beta_f - \alpha_f\eta_f - \ln \sum_{f\in\mathcal{F}} z_f. \tag{A33}$$

Since the log-sum-exp function $\ln \sum_{f\in\mathcal{F}} z_f$ is convex with respect to $\boldsymbol{\eta}$ (Boyd and Vandenberghe 2004), the $S_f(\boldsymbol{\eta})$ is log-concave with respect to $\boldsymbol{\eta}$ due to concavity additivity property. Besides, we have

$$\ln P_f(\boldsymbol{\eta}) = \ln\omega + \ln S_f(\boldsymbol{\eta}) + \ln(\eta_f + \mu_f - \delta_f\eta_0), \tag{A34}$$

the $P_f(\boldsymbol{\eta})$ is also log-concave with respect to $\boldsymbol{\eta}$ due to concavity additivity property.

Appendix 2. Iterative algorithm for solving optimal equilibrium conditions

The iterative algorithm is shown as the Algorithm 1. We assume that the market competition starts at the 0th iteration step, when every unit adopts its initial data setting. For convenience of representation, we denote $\eta_{f,0}$ as the initial gate fee and $\eta_{f,k}^\dagger$ as the optimal gate fee of treatment unit f at the kth iteration step, where $f \in \mathcal{F}$. Besides, we denote $\boldsymbol{\eta}_k^\dagger$ as the optimal gate fee vector and $\boldsymbol{\eta}_{f^-,k}^\dagger$ as the $\boldsymbol{\eta}_k^\dagger$ without the fth component $\eta_{f,k}^\dagger$ at the kth iteration step, where $f \in \mathcal{F}$. Further, we denote $\eta_{f,k}^*$, $\underline{\eta_{f,k}}$, $\tilde{\eta}_{0,k}$, $\eta_{0,k}^-$ and $\eta_{0,k}^\Gamma$ as the corresponding notations at the kth iteration step, where $f \in \mathcal{F}$. Moreover, we denote $\boldsymbol{\eta}^\infty$ as the optimal equilibrium gate fee vector and ξ as the stop criteria of the iterative algorithm. Finally, it should be noted that the Case (III) in the Step 4 has two gate fee setting options resulting in the same GTU payoff according to the Proposition 3.4. The decision-maker can choose preferred option according to his own preference.

Algorithm 1: Iterative algorithm for solving optimal equilibrium conditions.

Input: The initial data setting including $\eta_{f,0}$, ω, $\overline{\eta}_f$, α_f, β_f, μ_f and δ_f for all $f \in \mathcal{F}$.
Output: The optimal equilibrium gate fee vector $\boldsymbol{\eta}^\infty$.

1 Set $k = 0$ and $\eta_{f,k}^\dagger = \eta_{f,0}$ for $f \in \mathcal{F}$.

2 Set $k = k + 1$. For all $f \in \mathcal{I}$, calculate the $\eta_{f,k}^*$ by substituting $\boldsymbol{\eta}_{f^-,k-1}^\dagger$, ω, $\overline{\eta}_f$, β_f, α_f, μ_f and δ_f into the Equation (10). Based on the Proposition 3.1, check
(I) if $\eta_{f,k}^* \in (\overline{\eta}_f, \infty)$, set $\eta_{f,k}^\dagger = \overline{\eta}_f$;
(II) if $\eta_{f,k}^* \in (\underline{\eta_{f,k}}, \overline{\eta}_f]$, set $\eta_{f,k}^\dagger = \eta_{f,k}^*$.

3 Calculate the $\tilde{\eta}_{0,k}$ by substituting $\boldsymbol{\eta}_{0^-,k-1}^\dagger$, ω, $\overline{\eta}_f$, β_f, α_f, μ_f and δ_f into the Equation (A14). Based on the Proposition 3.4, check
(I) if $U(\tilde{\eta}_{0,k}|\boldsymbol{\eta}_{0^-,k-1}^\dagger) \geqslant 0$, set $\eta_{0,k}^\dagger = \overline{\eta}_0$ and go to *Step 5*;
(II) otherwise, go to *Step 4*.

4 Calculate $\eta_{0,k}^-$ and $\eta_{0,k}^\Gamma$ by substituting $\boldsymbol{\eta}_{0^-,k-1}^\dagger$, ω, $\overline{\eta}_f$, β_f, α_f, μ_f and δ_f into the model (12) and model (13), respectively. Based on the Proposition 3.4, check
(I) if $\overline{\eta}_0 \in (\underline{\eta_{0,k}}, \eta_{0,k}^-] \cup (\eta_{0,k}^\Gamma, \infty)$, set $\eta_{0,k}^\dagger = \overline{\eta}_0$;
(II) if $\overline{\eta}_0 \in (\eta_{0,k}^-, \eta_{0,k}^\Gamma)$, set $\eta_{0,k}^\dagger = \eta_{0,k}^-$;
(III) if $\overline{\eta}_0 = \eta_{0,k}^\Gamma$, set $\eta_{0,k}^\dagger = \overline{\eta}_0$ (Option 1) or $\eta_{0,k}^-$ (Option 2).

5 Check,
(I) if $|\eta_{f,k-1}^\dagger - \eta_{f,k}^\dagger| \leqslant \xi$ for all $f \in \mathcal{F}$, set $\boldsymbol{\eta}^\infty = \boldsymbol{\eta}_k^\dagger$ and terminate the algorithm;
(II) otherwise, go to *Step 2*.