A market competition model for waste recycling systems

Conference Paper · October 2016

Xiong Jie
National University of Singapore

Zhou HE
National University of Singapore

A MARKET COMPETITION MODEL FOR WASTE RECYCLING SYSTEMS

Tsan Sheng Adam Ng, Jie Xiong, Zhou He
Department of Industrial and Systems Engineering, National University of Singapore

Abstract: A waste recycling system is symbiotic when there exists physical exchange of material or by-products between different treatment units. We propose a mathematical model for studying the interactive behavior of different waste treatment operators in a symbiotic environment. We study the properties and gate fee strategies of different operators to discuss their behavior and the effects of various intervention strategies. We also propose a numerical algorithm to solve the model, yielding the optimal equilibrium gate fee charges, payoff and market share levels of different operators. Finally, computational studies based on a two-unit scenario in a case study of organic waste recycling is performed. Our results strongly suggest that, to improve new treatment technology utilization, subsidizing the operating cost of the new treatment unit is more effective in the long-run than exerting control on the gate fee upper bounds of the operators by the system regulator.

Keywords: Symbiotic waste recycling system; private sector participation; market competition; sustainability.

1. INTRODUCTION

Effective waste management continues to be an important priority in modern societies, the challenges of which include diminishing disposal space availability, harmful emissions, water and soil contaminations, and energy efficiency. Waste management is conventionally regarded as a public service, with the local government or authority having sole proprietorship of all entities and resources for the end-to-end activities from waste collection to final disposal. More recently, however, private sector participation in waste management is increasingly being studied and applied. Competition in waste management industry acts as an incentive for private operators to enhance their quality of waste management services, energy recovery efficiency and improve sustainability (Suocheng *et al.*, 2001). However, there are usually incompatibilities between a private operator's focus on short-term return on investment, with the state's long-term perspective needed to realize sustainability targets (Koppenjan *et al.*, 2009). Hence, the study on the role and action of private operators in the waste management industry is of interest and value.

In this work, we consider a scenario where treatment service providers (also termed *operators*), can compete in the waste recycling market by setting their preferred gate fee charge levels. On the other hand, the treatment service *users* (e.g. kerbside waste collectors) can choose their preferred service providers based on the price attractiveness. We also assume that all operators are self-interested entities that seek only to maximize their own respective payoffs. The operators interact with each other through gate fee competition for the market share of available input feedstock, and possibly also through co-treatment. In particular, we assume that some operators may require the service of other units to process their post-treatment residuals. Therefore, the relationships among the operators are not only competitive, but also *symbiotic* in nature. Such a scenario is motivated based on an actual case where the incumbent treatment approach for food waste is via conventional combustion. The environmental policy-makers are proposing a feasibility study of improving food waste recycling by introducing anaerobic digestion units in the townships. Independent and qualified operators can enter the market as digestion unit operators and offer their treatment services to the waste collectors. On the other hand, the digestion residuals require post-treatment by conventional combustion for the purpose of volume reduction before landfill disposal.

We model the symbiotic waste recycling system to study the qualitative behavior of the treatment unit operators. The model is solved by a proposed iterative algorithm to yield the optimal equilibrium conditions such as the respective gate fee charges, payoff levels and market share levels. The term *equilibrium* in this study refers to the state in which no operator has any more payoff to gain by changing its gate fee charge level. Some main findings of our model analysis are as follows. First, we show that in general, treatment units can exhibit different dominant operating regimes, depending on their gate fee levels. They may choose to operate in a market competition regime, by depressing gate fee charge to increase primary feedstock share. They can also exit the competition and focus on the residuals feedstock market. Consequently, the optimal gate fee strategy depends on the operator's gate fee attractiveness relative to the competition. The numerical study based on the Singapore case demonstrates that, to improve the utilization of new treatment technologies, provision of operational cost subsidies is more effective in the long-run, than simply depressing its gate fee upper bound due to the rebound effect of the market competition.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the problem description. Section 4 analyses the payoff properties and competitive behavior of waste recycling operators and system. Section 5 demonstrates the computational studies and Section 6 concludes this paper.

2. LITERATURE REVIEW

Relatively few work has modelled market competition in deregulated waste management systems. For example, Davila *et al.* (2005) proposed a two-tiered gray integer programming-based game theory approach to help scrutinize scenarios wherever landfills display competitive behavior under an increasing need for their services. The first tier is a gray integer programming model for minimizing the net system cost over one-year planning period by routing waste streams to two local landfill plants. The second tier is a series of gray integer sub-models with different landfill tipping fee strategies among competition for understanding the minute effect of landfill tipping fee changes on a city's optimal routing decision. Jørgensen (2010) studied a differential game among three neighboring regions in a finite time horizon, in which each region decides to dump a fraction of its waste stock to other regions for reducing the cost incurred by holding waste stock. Bárcena-Ruiz *et al.* (2015) studied a two-stage game between two private collection firms who pursue maximal payoffs by deciding their locations (first-stage decision) and prices (second-stage decision) when the government requires private collection firms or consumers to bear the waste transportation fees. It showed that regardless of whether two private collection firms' locations are chosen simultaneously or sequentially, the optimal location of waste collection point for maximizing weighted welfare of firms and consumers is in the middle of city. Besides, the optimal location of collection point is outside the middle of city only when the transportation fees are paid by consumers and the firms choose their locations sequentially.

Although a number of operations management models have been proposed for studying and optimizing waste management activities, to the best of the authors' knowledge, no prior work has established a model to study the behavior of waste treatment operators in a symbiotic waste recycling system under a market competitive gate fee charge environment. As remarked in Karmperis *et al.* (2013), a decision support model identifying the strategy that forms an equilibrium state could help enhance the sustainability of the entire waste management system. Besides, with the increasing trend of private sector participation in the waste management value chain, the market mechanism is playing an increasingly prominent role in the industry. In our work, we apply a utility-based market share model to imitate waste treatment service users' preferences based on gate fee charges. We model each waste treatment operator as self-interested players who are only concerned with maximizing their individual payoff functions. Waste treatment operators can also interact with each other via post-treatment activities. Based on such a model, we can compute the optimal equilibrium conditions and study various intervention strategies to achieve sustainability aspirations.

3. PROBLEM AND MODEL DESCRIPTION

3.1 Problem description and background

The general process of waste management is as follows. Mixed wastes are first collected and presorted by refuse collectors. Next, material recovery takes place for recyclable and re-usable items. The remaining wastes are then sent to, and treated via various approaches (such as waste-to-energy) for volume compression and useful energy recovery. Residuals generated may then require further post-treatment. Finally, all post-treatment residuals and other un-treatable components, are disposed in landfill sites. In this work, we focus on the waste recycling stage of the process as illustrated in Figure 1, which comprises of different technologies and units that can treat either selected streams of waste (e.g., anaerobic digestion for the treatment of organic waste), or general waste (e.g., incineration). The collected and presorted waste streams are collectively referred to as the *input feedstock*. The treatment service users (e.g. refuse collectors) forward the input feedstock to one or several of the treatment units, and pay a service tipping fee, termed as the *gate fee*, to the treatment unit operator. We also refer to treatment units that treat only specific waste stream components as *specialized treatment units (STUs)*, and those that can treat any waste stream component as *general treatment unit (GTU)*. In particular, the market competition model is built on the following assumptions:

(1) The waste streams considered in the input feedstock can be treated by all units in the system, so that the boundary of analysis is focused on waste stream components that are of interest to governing agencies, e.g. organic waste.

(2) Only one GTU is considered due to economies of scale.

(3) The STUs can produce post-treatment residuals that requires further processing by the GTU before its final disposal.

(4) Each treatment unit operator can freely determine his gate fee charge level, subjected to an upper bound imposed by state regulation, to maximize his own payoff.

(5) The landfilling process and the post-treatment process for airborne and waterborne emissions from all treatment units are excluded in the model.

3.2 Model description

The notation used in the model formulation are listed in the Table 1. The index value of ``0" denotes the GTU. Other notations are introduced as needed in this paper.

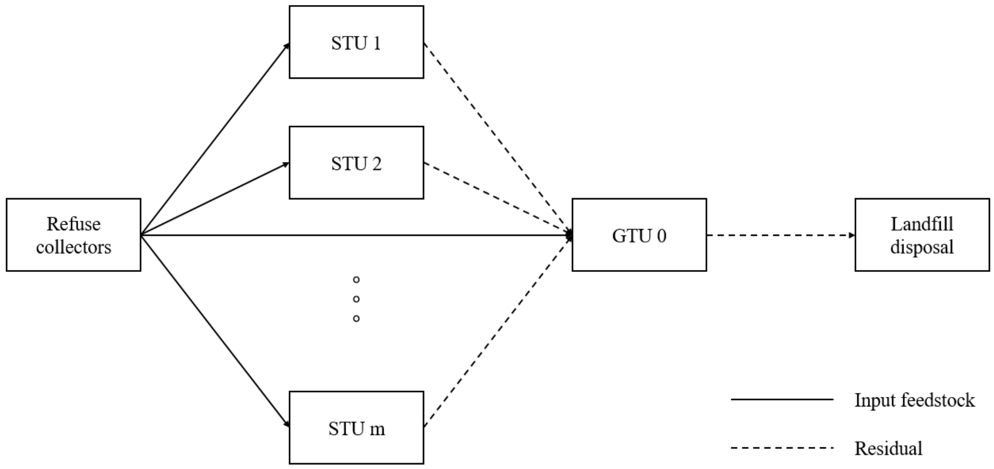

Figure 1. Topology diagram of the considered waste recycling process

Table 1. Notation in the decision support model

Notation	Definition
I	The set of STU, where $I = \{1,2,\dots,m\}$.
F	The set of all treatment units, where $F = \{0\} \cup I$.
η_f	Gate fee of the treatment unit f, where $f \in F$.
$\vec{\eta}$	Gate fee vector, i.e. $\vec{\eta} = (\eta_0, \eta_1, \dots, \eta_m)$.
η_f^{∇}	Optimal gate fee of the treatment unit f, where $f \in F$.
η_f^{∞}	Optimal equilibrium gate fee of the treatment unit f, where $f \in F$.
$\bar{\eta}_f$	Gate fee upper bound of the treatment unit f, where $f \in F$.
μ_f	Operation profit rate of the treatment unit f, where $f \in F$.
δ_f	Residual coefficient of the treatment unit f, where $f \in F$.
α_f	Customer price sensitivity to the treatment unit f, where $f \in F$.
β_f	Other comprehensive customer utility index of the treatment unit f, where $f \in F$.
ω	Total volume of collected input feedstock.
$S_f(\vec{\eta})$	Market share function of the treatment unit f, where $f \in F$.
$P_f(\vec{\eta})$	Payoff function of the treatment unit f, where $f \in F$.

We first introduce the market share function $S_f(\vec{\eta})$ to model the proportion of input feedstock received by the various treatment units, where $f \in F$. The market share function is based on the *multinomial logit demand* model (Guadagni *et al.*, 1983). In our model, the market share of input feedstock received by treatment unit f is given by

$$S_f(\vec{\eta}) = \frac{e^{\beta_f - \alpha_f \eta_f}}{\sum_{f \in F} e^{\beta_f - \alpha_f \eta_f}}, \forall f \in F, \tag{1}$$

where $\alpha_f > 0$ and β_f are parameters defining the function $\beta_f - \alpha_f \eta_f$ associated with treatment unit f, which can be regarded as a first-order approximation or surrogate for the `attractiveness' of using treatment unit f. We denote $\vec{\eta}_{f^-}$ as the gate fee vector $\vec{\eta}$ without the f^{th} component η_f for all $f \in F$. For each treatment operator f, it is also easy to see that his market share $S_f(\eta_f | \vec{\eta}_{f^-})$ is non-increasing in his gate fee η_f for all $f \in F$.

In the system considered, the STUs accrue revenues through the collection of gate fees for receiving input feedstock, as well as by selling recovered products such as power or energy (e.g. electricity or trucking fuel) from the treatment process. Its expenditures include variable treatment operation costs (e.g. transportation and process cost) and also the cost for using the service of the GTU for disposing its post-treatment residuals. This last component is essentially the gate fee charged by the GTU. For convenience, we denote μ_f as the operation profit rate from recovered product sales per unit less all cost components except for the residual post-treatment cost incurred as the form of the gate fee payment to the GTU. The payoff maximization model for a STU (namely $\forall f \in F$) is then defined as follows.

$$\max_{\eta_f} \omega\, S_f(\eta_f|\vec{\eta}_{\boldsymbol{f}^-})(\eta_f + \mu_f - \delta_f \eta_0) \tag{2}$$
$$s.t.\ \eta_f + \mu_f - \delta_f \eta_0 > 0 \tag{3}$$
$$\eta_f \le \bar{\eta}_f \tag{4}$$

Objective (2) represents the payoff function of the STU f, computed using the product of the total input feedstock received, $\omega S_f(\eta_f|\vec{\eta}_{\boldsymbol{f}^-})$, and profit rate $\eta_f + \mu_f - \delta_f \eta_0$. The parameter $\delta_f \in (0,1)$ denotes *residual coefficient*, which is the proportion per unit input feedstock received that requires post-treatment by the GTU. Constraint (3) ensures that the operator is only interested in positive marginal profits. Constraint (4) implies the gate fee of each STU should be less than a regulated upper bound. This gate fee upper bound is imposed to inhibit the socially unacceptable prices in waste services.

For the GTU, the operator's revenue stream comes from gate fee collected from both classes of service users, i.e. the refuse collectors, and the STUs, as well as selling recovered energy products. The considered expenditures of GTU include variable treatment operation costs and also final disposal service costs. The payoff maximization model for the GTU is stated as follows.

$$\max_{\eta_0}\left[\omega S_0(\eta_0|\vec{\eta}_{\boldsymbol{0}^-}) + \textstyle\sum_{f\in I} \omega S_f(\eta_0|\vec{\eta}_{\boldsymbol{0}^-})\, \delta_f\right](\eta_0 + \mu_0) \tag{5}$$
$$s.t.\ \eta_0 + \mu_0 > 0 \tag{6}$$
$$\eta_0 \le \bar{\eta}_0 \tag{7}$$

Objective (5) represents the payoff function of the GTU. Constraint (6) ensures the GTU has a positive marginal profit. Constraint (7) requires the gate fee of GTU to satisfy the corresponding government regulations as Constraint (4).

4. PAYOFF PROPERTIES AND GATE FEE STRATEGIES ANALYSIS

In this section, we study the payoff functions' properties for the two types of treatment units. To simplify the notation, we denote the payoff functions as $P_f(\vec{\eta}) = \omega S_f(\vec{\eta}_{\boldsymbol{f}})(\eta_f + \mu_f - \delta_f \eta_0),\ \forall f \in I$, and $P_0(\vec{\eta}) = \left[\omega S_0(\vec{\eta}) + \sum_{f\in I} \omega S_f(\vec{\eta})\,\delta_f\right](\eta_0 + \mu_0)$. In addition, we denote $\underline{\eta_f} = -\mu_f + \delta_f \eta_0$ for all $f \in I$, and $\underline{\eta_0} = -\mu_0$, which can be seen as the lower bound of the gate fee for treatment unit f to secure positive marginal payoff. We also denote η_f^{∇} as the optimal gate fee and denote η_f^{∞} as the optimal equilibrium gate fee, where $f \in F$. Specifically, η_f^{∇} is the gate fee that maximizes the $P_f(\vec{\eta})$ over the domain $(\underline{\eta_f}, \bar{\eta}_f)$, and η_f^{∞} is the η_f^{∇} under the equilibrium condition. Finally, we denote $z_f = e^{\beta_f - \alpha_f \eta_f},\ \forall f \in F$.

4.1 Properties of the STU's payoff function

We first consider the payoff function of STU, and its optimal gate fee strategy.

Proposition 4.1. *The payoff function $P_f\left(\eta_f|\vec{\eta}_{\boldsymbol{f}^-}\right)$ is log-concave with respective to η_f when $\eta_f > \underline{\eta_f}$ for all $f \in I$. Furthermore, the payoff maximizing gate fee η_f^* over the domain $(\underline{\eta_f}, \infty)$ can be obtained by solving the following equation:*

$$\eta_f^* - \underline{\eta_f} - \frac{z_f^* + \sum_{f'\in F/\{f\}} z_{f'}}{\alpha_f \sum_{f'\in F/\{f\}} z_{f'}} = 0,\ where\ z_f^* = e^{\beta_f - \alpha_f \eta_f^*}. \tag{8}$$

All technical proofs in this paper are provided in the Appendix A. Proposition 4.1 shows that there must exist a unique gate fee η_f^* for each STU to achieve its maximum payoff over the domain $(\underline{\eta_f}, \infty)$. The value of η_f^* can be obtained by numerical solvers. Regarding the impact of the gate fee upper bound $\bar{\eta}_f$ on the optimal gate fee η_f^{∇} in a STU, Proposition 4.1 implies that, if $\eta_f^* \le \bar{\eta}_f$, then $\eta_f^{\nabla} = \eta_f^*$. Otherwise $\eta_f^{\nabla} = \bar{\eta}_f$.

4.2 Properties of the GTU's payoff function

We next consider the payoff function of GTU, and its optimal gate fee strategy.

Proposition 4.2. *Define the function $U(\vec{\eta})$ as*

$$U(\vec{\eta}) = ln \sum_{f\in F} z_f + ln \sum_{f\in F} z_f\, \delta_f - ln\, z_0 - ln\left(\eta_0 - \underline{\eta_0}\right) - ln\, \alpha_0 \sum_{f\in I} z_f\,(1-\delta_f), \tag{9}$$

and the minimal point $\tilde{\eta}_0$ *with* $\partial U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0 = 0$*. So if* $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) \geq 0$*, the GTU's optimal gate fee* η_0^{∇} *should be set to its upper bound* $\bar{\eta}_0$*. Otherwise, the* η_0^{∇} *should be set to: (a)* $\bar{\eta}_0$*, when* $\bar{\eta}_0 \in (\underline{\eta_0}, \eta_0^-] \cup (\eta_0^{\tau}, \infty)$*; (b)* η_0^-*, when* $\bar{\eta}_0 \in (\eta_0^-, \eta_0^{\tau})$*; (c)* η_0^{τ} *or* η_0^-*, when* $\bar{\eta}_0 = \eta_0^{\tau}$*. The* η_0^- *is the solution of model (10):*

$$\min_{\eta_0\in(\underline{\eta_0},\infty)} \eta_0\,,\ \ s.t.\, U(\vec{\eta}) \leq 0, \tag{10}$$

and η_0^{τ} *is the solution of model (11):*

$$\min_{\eta_0\in(\eta_0^-,\infty)} \eta_0\,,\ \ s.t.\, ln\, P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-}) \geq ln\, P_0(\eta_0^-|\vec{\eta}_{\mathbf{0}^-}). \tag{11}$$

Proposition 4.2 provides the optimal gate fee strategy for the GTU, given a STU's gate fee setting. If $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) \geq 0$, the payoff function of GTU is always non-decreasing, and hence it is always optimal to set its gate fee as high as possible, i.e. set $\eta_0^{\nabla} = \bar{\eta}_0$. More interestingly, if $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) < 0$ and the η_0 is relatively high, saying above η_0^{τ}, the GTU operator exits the market competition regime, and profit margins are now dominated by treatment of residues from STUs. Because the STU has no alternative but to use the GTU's service for post-treating residues, the GTU can always maximize his benefits by charging a high premium for the post-treatment service. This property reveals that, it is necessary for the regulator to mandate a sensible upper bound on the gate fee charge to prevent escalating costs of waste recycling service. On the other hand, the GTU might not necessarily always choose the upper bound imposed as its optimal gate fee. In particular, whenever $\bar{\eta}_0 \in (\eta_0^-, \eta_0^{\tau})$, the operator prefers a strictly lower gate fee η_0^-. Therefore, Proposition 4.2 provides useful information for the regulator to choose an appropriate gate fee upper bound.

5. COMPUTATIONAL STUDIES

In this section, we perform numerical studies of the proposed symbiotic waste recycling system based on the case of Singapore. In the current practice, all food wastes are sent for incineration (i.e. GTU). The local environmental regulator has been making substantial effort to implement anaerobic digestion (i.e. STU) for treating food waste with better energy recovery efficiency, less environmental emission and landfill usage. However, under-utilizations issue led to the shutdown of the largest private anaerobic digestion company IUT Global in 2011. Therefore, a computational study providing decision support for sustaining the operation of a STU in a competitive gate fee charge environment is of significant importance for the success of improving environmental sustainability in Singapore.

The computational study is conducted for a two-unit scenario, i.e. with one STU and one GTU. The proposed iterative algorithm in the Appendix B is applied to simulate the competitive gate fee charge environment. The initial gate fee of STU and GTU is set to 100 SGD/ton and 80 SGD/ton respectively. And the initial gate fee upper bounds of both units are assumed to be 20 SGD/ton higher than their initial gate fee settings. Besides, we set $\mu_0 = -6.67$ SGD/ton and $\mu_1 = -53.63$ SGD/ton according to McCrea *et al.* (2009). Further, we set $\delta_1 = 0.31$ according to De Bere (2000), and $\delta_0 = 0$ as the incineration does not generate residues requiring post-treatment, except for the final disposal. Moreover, we assume that the multinomial logit model parameters are $\alpha_0 = \alpha_1 = 0.1$, and $\beta_0 = \beta_1 = 15$ for simplicity. Finally, we consider the organic wastes as the input feedstock, and set $\omega = 1{,}409{,}400$ tons based on Singapore's data in 2014.

5.1 Influence of STU gate fee upper bound

The computational study on the influence of STU's gate fee upper bound $\bar{\eta}_1$ in the two-unit scenario is presented in this section. Specifically, we reduce $\bar{\eta}_1$ from the initial 120 SGD/ton to 80 SGD/ton and perform the numerical simulation. Table 2 shows the final equilibrium states of the two units under different $\bar{\eta}_1$. It can be seen that both units' equilibrium gate fees (i.e. η_0^{∞} and η_1^{∞}) and equilibrium log-payoffs (i.e. $\ln P_0(\eta_0^{\infty}, \eta_1^{\infty})$ and $\ln P_1(\eta_0^{\infty}, \eta_1^{\infty})$) decrease when the $\bar{\eta}_1$ decreases, due to market competition effects. This indicates that the GTU is operating in the market competition regime for feedstock. It also implies that the policy of reducing the $\bar{\eta}_1$ has a positive social impact as it reduces costs borne by treatment service users. However, the equilibrium market share of STU $S_1(\eta_0^{\infty}, \eta_1^{\infty})$ when $\bar{\eta}_1 = 80$ is only 3.51% higher than that when $\bar{\eta}_1 = 120$. This suggests that depressing $\bar{\eta}_1$ is not very effective in improving the STU utilization. Furthermore, decreasing $\bar{\eta}_1$ can cause significant payoff slump of the STU, which indicates that this policy can adversely affect its economic feasibility.

Table 2. Performance comparison of STU and GTU under the optimal equilibrium conditions with different $\bar{\eta}_1$

Indicator	Unit	$\bar{\eta}_1$ (SGD/ton)					
		120	90	87.5	85	82.5	80
η_0^∞	SGD/ton	78.06	77.51	75.43	73.38	71.35	69.33
η_1^∞	SGD/ton	90.67	90	87.5	85	82.5	80
$S_0(\eta_0^\infty,\eta_1^\infty)$	%	77.91	77.72	76.97	76.17	75.31	74.4
$S_1(\eta_0^\infty,\eta_1^\infty)$	%	22.09	22.28	23.03	23.83	24.69	25.6
$\ln P_0(\eta_0^\infty,\eta_1^\infty)$	-	18.16	18.25	18.22	18.18	18.14	18.1
$\ln P_1(\eta_0^\infty,\eta_1^\infty)$	-	15.2	15.17	15.04	14.88	14.67	14.38

6.2 Influence of STU operation profit rate

We now vary the STU operation profit rate μ_1 in the simulation. Table 3 presents the results of the two-unit system under different μ_1. It can be seen that the equilibrium gate fees of both units (i.e. η_0^∞ and η_1^∞) decrease with increasing μ_1, and the equilibrium log-payoff of STU $\ln P_1(\eta_0^\infty,\eta_1^\infty)$ increases. This result indicates that increasing μ_1 has the effect of reallocating benefits from GTU to STU and the service users. Furthermore, the equilibrium market share of STU $S_1(\eta_0^\infty,\eta_1^\infty)$ increases with increasing μ_1, implying that it is an effective approach to improve its utilization. However, Table 3 also shows that increasing μ_1 erodes the profitability of GTU (i.e. $\ln P_0(\eta_0^\infty,\eta_1^\infty)$). Hence, the policy of increasing the μ_1 should be carefully implemented to balance the economic considerations of the GTU.

Table 3. Performance comparison of STU and GTU under the optimal equilibrium conditions with different μ_1

Indicator	Unit	μ_1 (SGD/ton)					
		-53.63	-48.63	-43.63	-38.63	-33.63	-28.63
η_0^∞	SGD/ton	78.06	72.89	67.94	63.26	58.9	54.91
η_1^∞	SGD/ton	90.67	84.39	78.26	72.29	66.55	61.06
$S_0(\eta_0^\infty,\eta_1^\infty)$	%	77.91	75.96	73.73	71.16	68.25	64.91
$S_1(\eta_0^\infty,\eta_1^\infty)$	%	22.09	24.04	26.27	28.84	31.75	35.09
$\ln P_0(\eta_0^\infty,\eta_1^\infty)$	-	18.16	18.17	18.07	17.97	17.87	17.76
$\ln P_1(\eta_0^\infty,\eta_1^\infty)$	-	15.2	15.31	15.43	15.56	15.7	15.85

6. CONCLUSION

In this paper, a model was developed for the analysis of the payoff of self-interested specialized and general treatment unit (STU and GTU) operators in a symbiotic waste recycling system under market competition. All operators in the market compete for input waste by setting their preferred gate fee charge levels. A comprehensive analysis was conducted to discuss the properties of different unit operators' payoff functions, and to determine their optimal gate fee strategies. Based on the real case of Singapore, a computational study in the two-unit scenario was conducted by applying the proposed iterative algorithm to solve the decision support model. The simulation results of the impact of system parameters showed that, although increasing the STU's operation profit rate under some specific conditions can help improve system's environmental sustainability performance at the equilibrium state, it may deteriorate the system's economic or social impacts. Besides, to improve new treatment technology utilization, subsidizing the operating cost of the new treatment unit is more effective in the long-run than exerting control on the gate fee upper bounds of the operators by the system regulator.

APPENDIX A – PROOFS

Proof of Proposition 4.1. For every $f \in I$, when $\eta_f > \underline{\eta}_f$, its log-payoff function is

$$\ln P_f\left(\eta_f|\vec{\eta}_{f^-}\right) = \ln\omega + \ln S_f\left(\eta_f|\vec{\eta}_{f^-}\right) + \ln(\eta_f - \underline{\eta}_f). \tag{12}$$

It can be seen that the first term of the above function is constant, and the third term is concave with respect to η_f. Therefore, we only need to prove the concavity of the log-function $\ln S_f\left(\eta_f|\vec{\eta}_{f^-}\right)$ with respect to η_f. Here, we have

$$\frac{\partial^2 \ln S_f(\eta_f|\vec{\eta}_{f^-})}{\partial \eta_f^2} = \frac{\alpha_f^2 z_f(z_f - \sum_{f\in F} z_f)}{(\sum_{f\in F} z_f)^2} \le 0. \tag{13}$$

Therefore, the log-function $\ln S_f(\eta_f|\vec{\eta}_{f^-})$ is concave with respect to η_f when $\eta_f > \underline{\eta_f}$, $\forall f \in I$. Since addition preserves concavity, the log-payoff function $\ln P_f(\eta_f|\vec{\eta}_{f^-})$ is concave with respect to η_f when $\eta_f > \underline{\eta_f}$, $\forall f \in I$. Based on the concavity property, to determine the payoff maximizing gate fee η_f^* over the domain $(\underline{\eta_f}, \infty)$, we consider

$$\frac{\partial \ln P_f(\eta_f|\vec{\eta}_{f^-})}{\partial \eta_f} = \frac{1}{\eta_f - \underline{\eta_f}} - \frac{\alpha_f \sum_{f'\in F/\{f\}} z_{f'}}{\sum_{f\in F} z_f}. \tag{14}$$

It can be seen that if η_f approaches to $\underline{\eta_f}$, the value of $\partial \ln P_f(\eta_f|\vec{\eta}_{f^-})/\partial \eta_f$ will approach positive infinity. Besides, if η_f goes to positive infinity, the value of $\partial \ln P_f(\eta_f|\vec{\eta}_{f^-})/\partial \eta_f$ will approach $-\alpha_f$, which is less than 0. It means that $\ln P_f(\eta_f|\vec{\eta}_{f^-})$ is neither a monotonically increasing or decreasing concave function when $\eta_f > \underline{\eta_f}$. Hence, η_f^* must satisfy the condition $\partial \ln P_f(\eta_f^*|\vec{\eta}_{f^-})/\partial \eta_f = 0$, namely the equation (8).

Proof of Proposition 4.2. When $\eta_0 \in (\underline{\eta_0}, \bar{\eta}_0]$, denoting that $\delta_0 = 1$ and $z_0 = z_0\delta_0$ we have

$$\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-}) = \ln \omega + \ln \frac{\sum_{f\in F} z_f \delta_f}{\sum_{f\in F} z_f} + \ln(\eta_0 - \underline{\eta_0}). \tag{15}$$

Therefore, the first-order derivative of $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ with respect to η_0 is

$$\frac{\partial \ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})}{\partial \eta_0} = \frac{\sum_{f\in F} z_f\delta_f \sum_{f\in F} z_f - \alpha_0 z_0(\eta_0 - \underline{\eta_0})(\sum_{f\in I} z_f - \sum_{f\in I} z_f\delta_f)}{(\eta_0 - \underline{\eta_0})\sum_{f\in F} z_f\delta_f \sum_{f\in F} z_f}. \tag{16}$$

It can be seen that the sign of the function $\partial \ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0$ is only determined by its numerator as its denominator is definitely positive. From the definition of function $U(\vec{\eta})$, it is easy to know that: (a) when $U(\eta_0|\vec{\eta}_{\mathbf{0}^-}) > 0$, then $\partial \ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0 > 0$; (b) when $U(\eta_0|\vec{\eta}_{\mathbf{0}^-}) = 0$, then $\partial \ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0 = 0$; (c) when $U(\eta_0|\vec{\eta}_{\mathbf{0}^-}) < 0$, then $\partial \ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0 < 0$. Furthermore, when $\eta_0 > \underline{\eta_0}$, the convexity of $U(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ can be easily proved by applying the second-order condition and the convexity additivity property. Then we have

$$\frac{\partial U(\eta_0|\vec{\eta}_{\mathbf{0}^-})}{\partial \eta_0} = \frac{\alpha_0(\sum_{f\in F} z_f \sum_{f\in F} z_f\delta_f - z_0\sum_{f\in F} z_f\delta_f - z_0\sum_{f\in F} z_f)}{\sum_{f\in F} z_f \sum_{f\in F} z_f\delta_f} - \frac{1}{\eta_0 - \underline{\eta_0}}. \tag{17}$$

It can be seen that if η_0 approaches the $\underline{\eta_0}$, the value of $\partial U(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0$will approach negative infinity. Besides, if η_0 goes to positive infinity, the value of $\partial U(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0$ will approach α_0, which is greater than 0. Therefore, the $U(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is neither a monotonically increasing nor a monotonically decreasing convex function. Therefore, the value of $\tilde{\eta}_0$is unique and can be calculated by letting $\partial U(\eta_0|\vec{\eta}_{\mathbf{0}^-})/\partial \eta_0 = 0$. Based on the value of $\tilde{\eta}_0$, we discuss the three cases of the graph of $U(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ when $\eta_0 > \underline{\eta_0}$, which are shown in the Figure 2. In the Case I, we have $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) > 0$ for all $\eta_0 \in (\underline{\eta_0}, \bar{\eta}_0]$, which indicates that the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is increasing in η_0. Therefore, the η_0^∇ should be set to $\bar{\eta}_0$. In the Case II, we have $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) = 0$, which indicates that the $U(\eta_0|\vec{\eta}_{\mathbf{0}^-}) \ge 0$ for all $\eta_0 \in (\underline{\eta_0}, \bar{\eta}_0]$. Therefore, the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is non-decreasing in η_0, and the η_0^∇ should be set to $\bar{\eta}_0$. In the Case III, we have $U(\tilde{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) < 0$. Then, there must exists two gate fees that let $U(\eta_0|\vec{\eta}_{\mathbf{0}^-}) = 0$. Here, we denote them as $\eta_0^-$and η_0^+, where $\eta_0^- < \eta_0^+$. Without considering the impact of $\bar{\eta}_0$ first, it can be seen that the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is decreasing in η_0 only when $\eta_0 \in (\eta_0^-, \eta_0^+)$, and increasing in η_0 when $\eta_0 \in (\underline{\eta_0}, \eta_0^-) \cup (\eta_0^+, \infty)$.

Now we consider the impact of $\bar{\eta}_0$ on the choice of η_0^∇ in the Case III. If $\bar{\eta}_0 \in (\underline{\eta_0}, \eta_0^-]$, the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is increasing in η_0, which indicate that the η_0^∇ should be set to $\bar{\eta}_0$. If $\bar{\eta}_0 \in (\eta_0^-, \eta_0^+]$, the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is increasing in η_0 when $\eta_0 \in (\underline{\eta_0}, \eta_0^-]$, but decreasing in η_0 when $\eta_0 \in (\eta_0^-, \bar{\eta}_0]$. It indicates that the η_0^∇ should be set to η_0^-. If $\bar{\eta}_0 \in (\eta_0^+, \eta_0^\tau)$, the $\ln P_0(\eta_0|\vec{\eta}_{\mathbf{0}^-})$ is increasing in η_0 when $\eta_0 \in (\eta_0^+, \bar{\eta}_0]$. However, since the model (11) indicates that $\ln P_0(\eta_0^-|\vec{\eta}_{\mathbf{0}^-}) = \ln P_0(\eta_0^\tau|\vec{\eta}_{\mathbf{0}^-})$, we have $\ln P_0(\bar{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) < \ln P_0(\eta_0^-|\vec{\eta}_{\mathbf{0}^-})$. Thus the η_0^∇ should be set to η_0^-. If $\bar{\eta}_0 = \eta_0^\tau$, we have $\ln P_0(\bar{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) = \ln P_0(\eta_0^\tau|\vec{\eta}_{\mathbf{0}^-}) = \ln P_0(\eta_0^-|\vec{\eta}_{\mathbf{0}^-})$, and the η_0^∇ can be set to either η_0^- or $\bar{\eta}_0$. If $\bar{\eta}_0 \in (\eta_0^\tau, \infty)$, we have $\ln P_0(\bar{\eta}_0|\vec{\eta}_{\mathbf{0}^-}) > \ln P_0(\eta_0^-|\vec{\eta}_{\mathbf{0}^-})$. Thus the η_0^∇ should be set to $\bar{\eta}_0$.

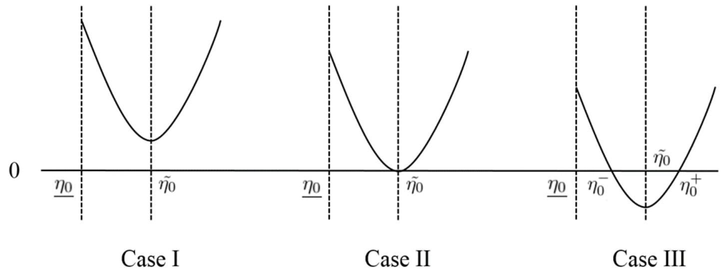

Figure 2. The graphs of three cases of the function $U(\eta_0|\vec{\eta}_{\mathbf{0}^-})$

APPENDIX B – ITERATIVE ALGORITHM FOR SOLVING OPTIMAL EQUILIBRIUM CONDITIONS

The iterative algorithm is based on the Proposition 4.1 and 4.2. We assume that the competition starts at the 0^{th} iteration step. For convenience, we denote $\eta_{f,0}$ as the initial gate fee and $\eta_{f,k}^{\nabla}$ as the optimal gate fee of treatment unit f at the k^{th} iteration step. Besides, we denote $\vec{\eta}_{f^-,k}^{\nabla}$ as the optimal gate fee vector $\vec{\eta}_k^{\nabla}$ without the f^{th} component $\eta_{f,k}^{\nabla}$ at the k^{th} iteration step. Further, we denote $\eta_{f,k}^*$, $\underline{\eta}_{f,k}$, $\tilde{\eta}_{0,k}$, $\eta_{0,k}^-$ and $\eta_{0,k}^{\tau}$ as the corresponding notations in the Table 1 at the k^{th} iteration step. We denote $\vec{\eta}^{\infty}$as the optimal equilibrium gate fee vector and ε as the stop criteria. The algorithm is shown as below.

Step 1: Input the initial data, including ω, $\overline{\eta}_f$, α_f, β_f, μ_f and δ_f for all $f \in F$. Set $k = 0$ and $\eta_{f,k}^{\nabla} = \eta_{f,0}$ for all $f \in F$.

Step 2: Set $k = k + 1$. For all $f \in F$, calculate the $\eta_{f,k}^*$ by substituting $\vec{\eta}_{f^-,k-1}^{\nabla}$ and the initial data into the equation (8). Check, if $\eta_{f,k}^* \in (\overline{\eta}_f, \infty)$, set $\eta_{f,k}^{\nabla} = \overline{\eta}_f$; if $\eta_{f,k}^* \in (\underline{\eta}_{f,k}, \overline{\eta}_f]$, set $\eta_{f,k}^{\nabla} = \eta_{f,k}^*$.

Step 3: Calculate the $\tilde{\eta}_{0,k}$ by applying the initial data into the equation $\partial U(\tilde{\eta}_{0,k} \mid \vec{\eta}_{f^-,k-1}^{\nabla})/\partial\eta_0 = 0$. Check, if $U(\tilde{\eta}_{0,k} \mid \vec{\eta}_{f^-,k-1}^{\nabla}) \geq 0$, set $\eta_{0,k}^{\nabla} = \overline{\eta}_0$, and go to Step 5; otherwise, go to Step 4.

Step 4: Calculate the $\eta_{0,k}^-$ and $\eta_{0,k}^{\tau}$ by applying the $\vec{\eta}_{f^-,k-1}^{\nabla}$ and the initial data in to the model (10) and (11), respectively. Check, if $\overline{\eta}_0 \in (\underline{\eta}_{0,k}, \eta_{0,k}^-] \cup (\eta_{0,k}^{\tau}, \infty)$, set $\eta_{0,k}^{\nabla} = \overline{\eta}_0$; if $\overline{\eta}_0 \in (\eta_{0,k}^-, \eta_{0,k}^{\tau})$, set $\eta_{0,k}^{\nabla} = \eta_{0,k}^-$; if $\overline{\eta}_0 = \eta_{0,k}^{\tau}$, set $\eta_{0,k}^{\nabla} = \eta_{0,k}^-$ or $\eta_{0,k}^{\tau}$ based on decision maker's preference.

Step 5: Check, if $|\eta_{f,k-1}^{\nabla} - \eta_{f,k}^{\nabla}| \leq \varepsilon$ for all $f \in F$, set $\vec{\eta}^{\infty} = \vec{\eta}_k^{\nabla}$ and terminate the algorithm; otherwise, go to Step 2.

REFERENCES

Bárcena-Ruiz, J. C., & Casado-Izaga, F. J. (2015). Regulation of waste management under spatial competition. *Journal of Cleaner Production*, 92, 216-222. Chicago

Davila, E., Chang, N. B., & Diwakaruni, S. (2005). Landfill space consumption dynamics in the Lower Rio Grande Valley by grey integer programming-based games. *Journal of Environmental Management*, 75(4), 353-365. Chicago

De Bere, L. (2000). Anaerobic digestion of solid waste: state-of-the-art. *Water science and technology*, 41(3), 283-290.

Jørgensen, S. (2010). A dynamic game of waste management. *Journal of Economic Dynamics and Control*, 34(2), 258-265. Chicago

Karmperis, Athanasios C., Konstantinos Aravossis, Ilias P. Tatsiopoulos, and Anastasios Sotirchos. "Decision support models for solid waste management: Review and game-theoretic approaches." *Waste management* 33, no. 5 (2013): 1290-1301.

Koppenjan, J. F., & Enserink, B. (2009). Public–private partnerships in urban infrastructures: reconciling private sector participation and sustainability. *Public Administration Review*, 69(2), 284-296.

McCrea, M., Tong, K. T., Hui-Hsuan, T., & Xiaoli, Z. 2009. A cost-benefit analysis of different waste-to-energy technologies for the management of municipal solid waste in Singapore. Accessed April 8, 2015, http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.539.2717.

Suocheng, D., Tong, K. W., & Yuping, W. (2001). Municipal solid waste management in China: using commercial management to solve a growing problem. *Utilities Policy*, 10(1), 7-11.